Poleni´s Manuscripts about the Dome of Saint Peter's

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In 1742, the dome of Saint Peter's, at Rome, showed severe cracks. The alarm was increased until the Pope Benedict XIV decided to study the stability of the dome and a possible way to reinforce it. He asked three famous mathematicians, Boscovich, Le Seur and Jacquier to write a technical report about the dome. Their *Parere* was published at the beginning of 1743. According to their calculations the dome was in a very dangerous condition and they advised to put iron rings around it and to reinforce the broken counterforts of the drum. Other scholars, alarmed about these conclusions, published their own reports about the *Parere* and the stability of the dome, which many of them found very safe. Amidst this of controversy, Benedict XIV decided to consult another famous Italian scholar, Giovanni Poleni.

Poleni was born in Venice in 1683. With his father he began to study Cartesian mathematics and the new experimental science. In 1710 he was named a member of the Royal Society in London. In 1739 he became part of the Faculty of Experimental Philosophy at the University of Padova and the following year he inaugurated the Theatre of Experimental Philosophy, a laboratory for experiments of the most diverse kinds. Later, in 1743 he received the invitation from Benedict XIV to make a report on the stability of the dome of St. Peter's. So, Poleni wrote two manuscripts in 1743 in which he explained the origin of the cracks and the way to increase the safety of the dome. He and also travelled to Rome (Di Stefano 1980 (1963); Brusatin 1971; López 1998b, 2001; Mainstone 2003). Through his collaboration with Vanvitelli in the placing of the tension rings he would prepare the *Memorie,* published in 1748, where he included all the information about the cracked dome and its restoration, plus summaries of the nearly thirty reports that were published during this period and an equilibrium analysis of the dome shell where he applied the safe theorem of the Limit Analysis for the first time (Straub 1952; Brusatin 1971; Heyman 1988, 1989, 1995, 1999 (1995), 2004 (1998); Mainstone 1989, 2003; Benvenuto 1991; Di Pasquale 1994; Huerta 1990, 1996, 2004; López 1998b, 2001). Later, he was to be put in charge of the work to restore the church of St. Anthony in Padova, destroyed in a fire, and about which there remains a manuscript. Also from 1749 there is a manuscript on the buildings of the Rialto. In 1756 a storm destroyed the roof of the main hall of the Palazzo della Ragione in Padova and, together with Colombo, Poleni was charged with its restoration. In 1759 he published a report on the facade of the Tower of St. Mark's in Venice. He finally died in Padova in 1761 (Salandin and Pancino 1987, pp. 15-9).

RIFLESSIONI DI GIOVANNI POLENI

The first report issued by Poleni was a manuscript signed on 21 March 1743 in Padova: "Riflessioni di Giovanni Poleni sopra i danni, e sopra la ristaurazione della cupola del Tempio di

San Pietro". In the introduction, Poleni already shows signs of the focus he will adopt, not just then, but throughout the time he would be involved in the case of the dome of St. Peter's. He considers that solidity is the principal quality that a work of architecture should have and also that the interrelation between theory and practice is of great importance in attacking architectural problems.

Figure 1. Anonymous portrait of Poleni at the Biblioteca Marciana in Venice (Salandin and Pancino 1987)

Poleni also admitted to never having seen the dome of St. Peter's. He had no other recourse but to base his research on books and drawings, which he does not specify, and, above all, on the information in the *Parere* (fig.2) and the *Risoluzione del Dubbio*.

THE INTERPRETATION OF THE DAMAGE IN THE *PARERE*

In the *Parere*, the three mathematicians prepared a geometrical model that would explain the movement of the dome which had caused the damage, and which at the same time would serve to evaluate the stability by applying the laws of mechanics, (fig.2). That model showed that each rib with its corresponding part of the dome, had subsided at its upper end, opening at the lower end due to the sagging of the drum and the buttresses. Poleni's fundamental point of disagreement with the three mathematicians was concerning this interpretation.

In the first place, Poleni could not understand how in this movement, the part NIHM, "an enormous amount of stones and cement" had remained intact (Riflessioni 1743, p.7 art.9). Which means that the similarity of the wooden stick that slides between a vertical plan and one that is horizontal also used by the three mathematicians and shown in (fig.3), cannot be valid. Neither are the conditions at the extremes of the wooden stick the same as in the dome, nor, above all, is the strength of the wood facing the masonry structure.

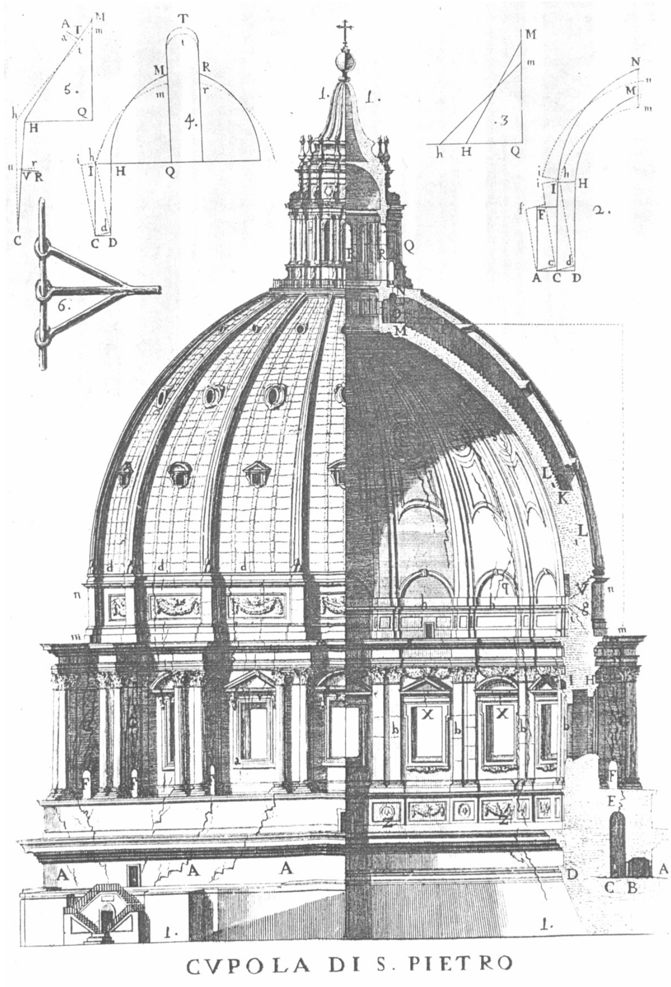

Figure 2. Cracked section and structural analysis of the dome of Saint Peter's
(Le Seur, Jacquier and Boscovich 1743)

Neither did Poleni agree that the double shell vault had suffered the same displacement as the ribbing. In other words, the model would be valid for a section of the dome between two planes close together, but not for larger sections. He reached this conclusion not only through a process of thought but:

> Following my inclination to experiment... I have ordered a small model of the Drum and of the Vault of the Dome to be made, which I have divided perpendicularly into four parts and fitted them together as indicated by the proposed system.
>
> (Riflessioni 1743, p.10 art.11)

Poleni´s model is shown in the his Plate 1 (fig.3). As can be seen, the movement described in the *Parere* could not have occurred without producing separations between the sections, both inside and outside, horizontally and vertically. The most important separation is the vertical uGFDABSe, the maximum width of which is found at the base of the dome. For the three-dimensional model built by Poleni to show that of the *Parere* to be correct, the damage to the real dome should coincide with that of the geometrical model, but this does not clearly occur. The three mathematicians talk of a dilatation in the diameter of the dome of approximately 24 ounces, but the vertical fissures do not always go all the way through the dome, from the inside to the outside, nor do they extend from the lantern at the base of the drum in all cases.

At the same time, analysing the horizontal fissures, the model in his Plate 2 (fig.3), can only be valid for an isolated rib. In reality, the rotation of the parts in one sector occurs around the points T and V, at the base of the drum, and the horizontal axes LF and BM, Fig.1 (fig.3), according to a mechanism that is more complex than that in Fig.2 (fig.3), where the rotation of the lower part of the dome seems to occur around a single point H. This would produce horizontal separations at the base of the dome and at the base of the drum that could not be observed in the real dome. (In this matter, Poleni also makes good use of his experience in various restoration works and cites the case of one of the domes of the Basilica of St. Mark's in Venice and that of the church of St. Anthony in Padova, to underline the different degree of seriousness posed by horizontal fissures compared to those that run vertically).

Even supposing that the dome had moved in this way, it would have caused its collapse, as in fact occurred with the model. And, if it had been kept standing by sealing the fissures, there would have been signs of these; the dome would have transmitted forces to the old tension rings, and horizontal cracks would have resulted, but none of these effects were produced. Poleni even spoke of the doubts of the three mathematicians over their own model. In conclusion, since there was no full coincidence in the real dome with the damage in the model, there was no justification for it.

THEORY OF ARCHES, VAULTS AND DOMES

The mathematical analysis of the model undertaken in the *Parere*, however, received no criticism from Poleni. Both the evaluation of the weights and of the contribution of the stability of each of them through the Virtual Work Principle appeared correct to him. However, as they were based on an erroneous model, Poleni thought they could be considered as valid (He will use three mathematicians´ evaluation of the weights in his *Memorie*, Di Pasquale 1994; López 1998b. About a reconstruction of the analysis included in the *Parere*, see Como 1997; López 1998, 1998b, 2001; Mainstone 2003).

On the other hand, in this part, two questions appear that were to be broadly developed in the *Memorie*, as an *a posteriori* reflection: the state of knowledge on the theory of arches, vaults and domes, and the resistance of the tension rings. The *Parere* spoke of La Hire and Couplet; to these Poleni adds in this first manuscript the contributions of Blondel, Parent, Frézier, Dulacq and Stirling (later he will mention Gregory, but he never quotes Hooke, even in his *Memorie*, López 1998b, p.438. About the state of knowledge on the theory of arches, vaults and domes see Benvenuto 1991; Heyman 1995, 1999 (1995), 2004 (1998); Huerta 1990, 1996, 2004; López 1998b).

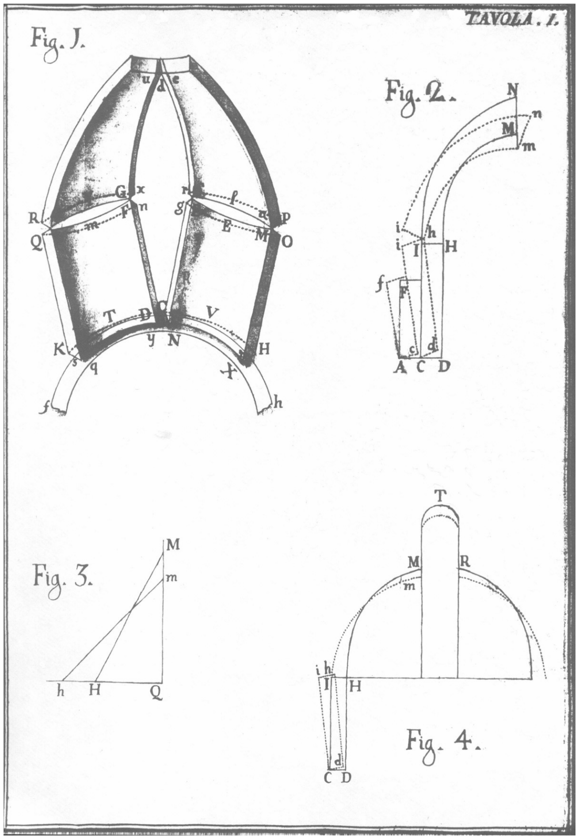

Figure 3. Analysis of the structural model proposed by Le Seur, Jacquier and Boscovich in their *Parere* (Riflessioni 1743, pl. I)

Regarding the resistance of the tension rings, Poleni talked of the contributions of Musschenbroek and his experiments on iron, and also cites Borelli and Mead. In terms of the capacity for resistance of ring-shaped iron, the name of Johann Bernoulli appears, but above all it is his own writings, dating from 1724, that he uses to "explain the tension of the cells forming the fibres of muscles"(Riflessioni 1743, p.26 art.28. See fig.5). As in the *Parere*, Poleni states that the relationship between the load supported by a straight rod and another in ring shape is approximately that existing between the radius and the perimeter of the circumference.

However, Poleni´s focus was to be eminently practical in the interpretation of the damage and considered that its cause resulted from the great weight of the dome:

> Although I am convinced that in Architecture one can very frequently use Mathematics to build structures, nevertheless, in cases when the structures may tend to a greater lesser degree to ruin, one should reason in a different fashion, and principally on the facts...it is too difficult and uncertain trying to speculate with a given theory over the causes. ..
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> (Riflessioni 1743, p.28 art.30)

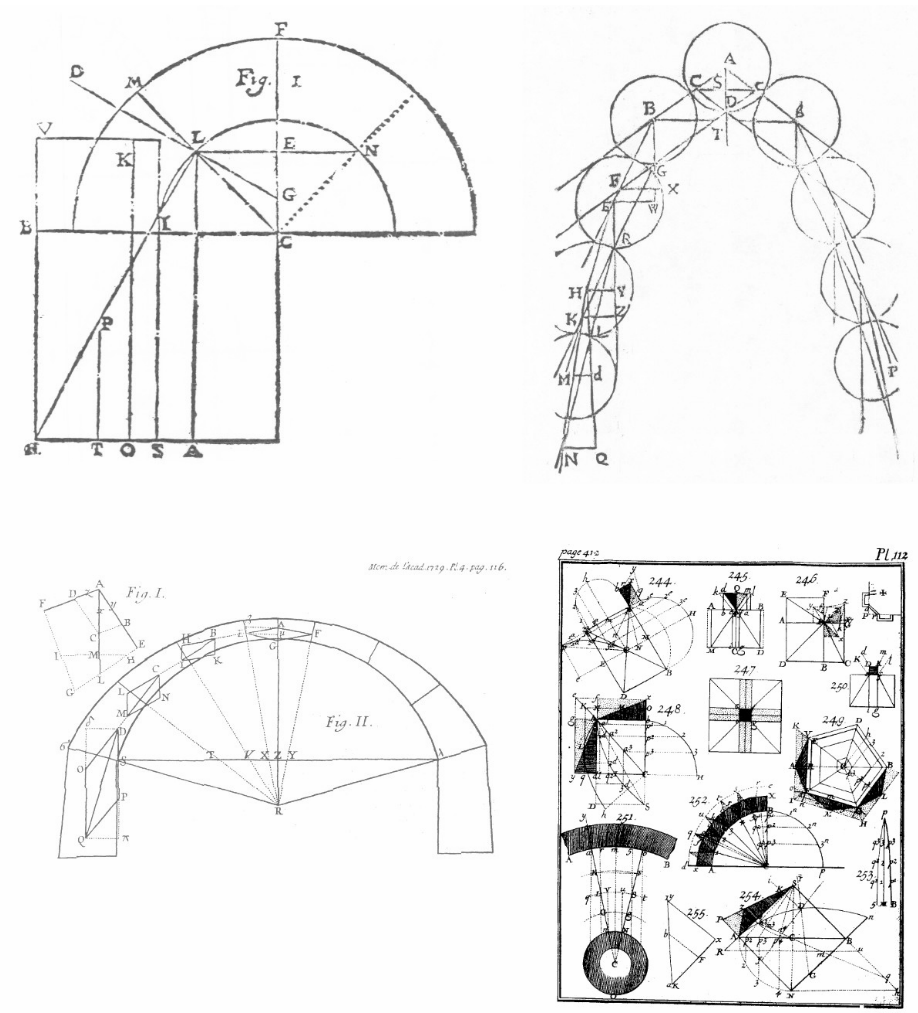

Figure 4. Theory of arches, vaults and domes: La Hire (1712), Stirling (1717), Couplet (1729), Frézier (1737-39) (from top left to bottom right)

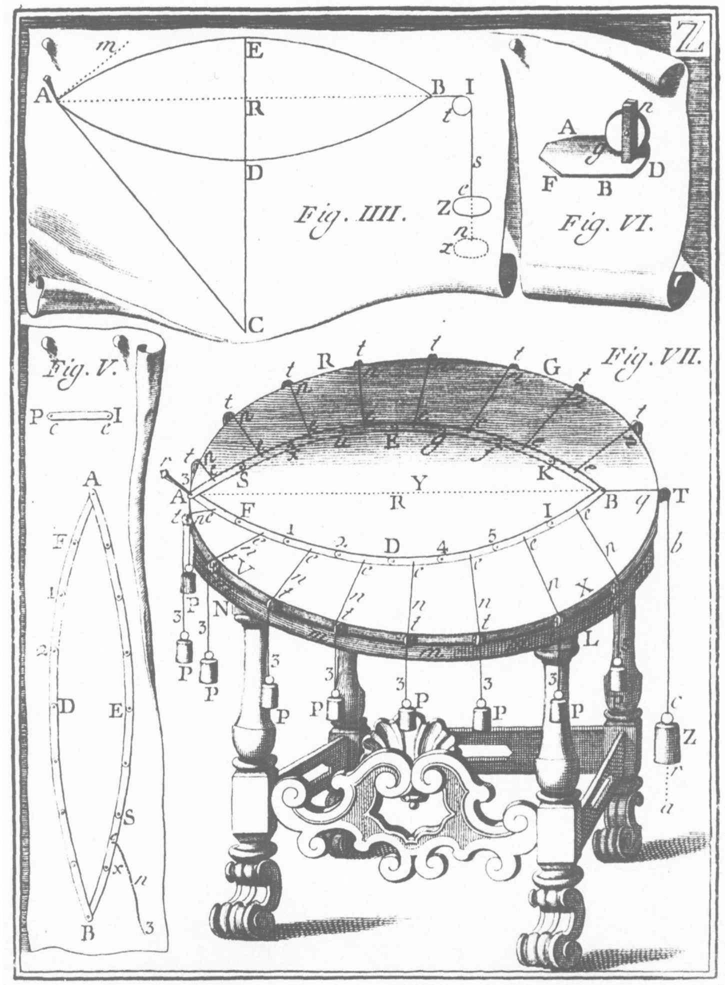

Figure 5. Tests for studying muscle movements (Poleni, 1729)

INTERPRETATION OF THE DAMAGE ACCORDING TO POLENI

Poleni starts by evaluating the importance of the damage according to the description in the *Parere* and the *Risoluzione*. Neither the three mathematicians nor Santini considered that the piers had moved or suffered damage, but Poleni thought that it was possible that there were small, undiscovered faults. He also considered significant in relation to this the affirmation of the *Parere* that the greatest damage in the dome was to be found over the piers, or that the drum in the area closest to them relied on a false support from the pendentives. In terms of the main arches, Poleni agreed with Santini that one should consider the damage to them, which was subsequently transmitted to the drum and the buttresses.

For analysing the causes of the damage, not observed personally, Poleni was to put his own observations and experience in restoration work to good use:

> From the observations which I have been able to make on several occasions, of the stones, cement and others, and of the methods of work...in building new structures or in demolishing or restoring them, I have thought it to be the diversity of accidents, which have further convinced me not to marvel that in some structures, even if not apparent (at least to the naked eye) caused from the beginning by their cement and the sagging of the walls, numerous cracks are produced... One can consider that the natural variations in the resistance of the stones, the faults in their workmanship, the different strength and solidity of the cement, the different constitution of the mortar of the same between the stones, causes the upper parts not to adapt and join, in all parts, perfectly with the lower parts, and that neither the compression of these parts, nor the connection between them, are equal, so that the weight, acting continuously (and which, as they say, never sleeps) on certain occasions comes apart and breaks the stones where the union is more defective or the strength of the coherence and the join is weaker.
>
> (Riflessioni 1743, pp.34-5, art.36)

It is to this cause then that he attributed the damage to the different parts of the dome, whether the arches, drums or shell: the constant pressure of the weight on faulty masonry. However, he did not consider the natural subsidence in masonry structures, consequence of defects in the stone and "contraction in the cement" (Riflessioni 1743, p.36, art.38) and explained that "…it was only after a lengthy period that large scale structures were affected by subsidence" (Riflessioni 1743, p.38, art.39).

What is specifically mentioned in terms of the drum is the materials bond. Due to the thickness of the walls and the negligence of the builders, it was likened to "that technique used by the Ancients which was referred to as *Murare a Cassa*" (Riflessioni 1743, p.36, art.37. See fig.6). The pressure exerted as a result of such a weight creates faults inside the walls, which in turn leads to more visible signs of damage.

In terms of the buttresses, Poleni explains the cause of the damage in Fig.5 (fig.7). Here it is clear to see how the wall and *eBDn* of the drum receive the full weight of the dome, as opposed to the *Apen* buttress which receives none. Accordingly,

> with the initial subsidence of the structure, in addition to the intense and constant compression resulting from the pressure on the Drum's wall as opposed to the buttress, it would be safe to assume that the latter would have given in to a greater extent than the former, thus producing cracks in one and not the other.
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> (Riflessioni 1743, p.39, art.40)

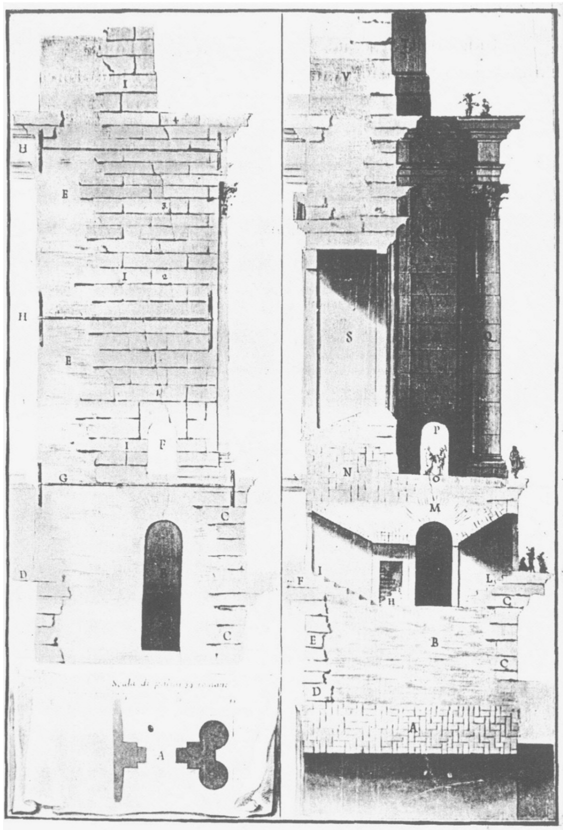


Figure 6. Details of the base, drum and counterforts of the dome of Saint Peter's and proposal to join the counterforts to the drum with iron bars by Vanvitelli (Di Stefano 1973)

The diagram also illustrates how the crack is produced in the *ctV* due to the fact that it is an area of lower density than that of the pilasters *pA* and *en*, and how it is weakened by arch *sc*. Added to this is the fact that the buttress is joined to the lower part of the base, which means that the crack would widen towards the upper, unencumbered section.

Furthermore, Poleni points to a second cause of the damage to the buttresses, the subsidence of the drum caused by the main arches, which the buttresses were unable to support.

Finally, Poleni speaks of the bending of the drum's pilasters. The three mathematicians claim that the outer pilasters had not curved outwards but had remained perpendicular, even inclining inwards, leading them to suspect that the structure might have purposely been built in such a way. This made Poleni doubt the claim that the inner pilasters had warped outwards, which would have constituted a serious oversight in Vitruvio's design.

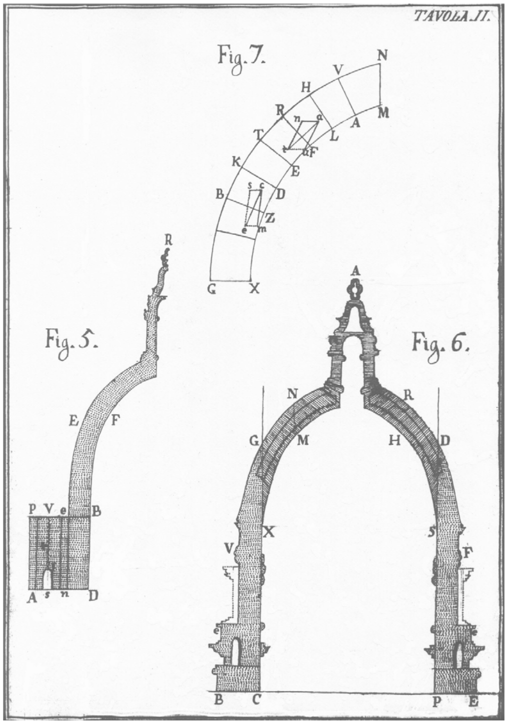


Figure 7. The stability of vaulted structures. Cross sections of the dome of Saint Peter's (Riflessioni 1743, pl. II)

STABILITY OF THE DOME

In order to assess the damage to the dome proper, Poleni posits a series of principles relating to the stability of the vaulted structures, while pointing out that the damage is the same as that sustained by the drum and buttresses. Poleni begins by identifying those parts of the structure which can be supported by those placed immediately below, perpendicularly and those supported by other means, in Fig.6 (fig.7). As can be observed, the relationship between some parts and others favours those which are self-supporting, separated by the GX and DS plans from those exerting outward thrusts, which means that weight is still exerted on the main arches, pendentives and piers. The upper sections, GNAMX and DRAHS, are therefore among those whose vaulted structures remain to be studied:

> The more they incline towards a horizontal position, the more the tendency would be for them to fall, although this should never actually happen: this inclination also leads to harmful outer thrusts, which clearly should have tended towards G and D, rather than between VX and FS.
>
> (Riflessioni 1743, pp.44-5, art.45)

Subsequently, in order to understand the behaviour of the vaulted section, one has to work according to the hypothesis that, whatever the material, it consisted of voussoirs and wedges, and so "what is said of the parts of an arch, must equally apply to the vault and dome" (Riflessioni 1743, p.45, art.46). Therefore, as can be seen in Fig.7 (fig.7), these wedges support one another in order to prevent a collapse as a result of the forces of gravity, and this thanks to its converging ledges. He again quotes Stirling, who illustrated the geometric form in which equilibrium is maintained between spheres of the same size, Fig.8 (fig.8) y (fig.4) to form an arch or a vault, with gravity as the only intervening force. Later Stirling deduces that an infinite reduction in the size of the spheres would lead to the catenary BGAH, Fig.9 (fig.8), which would be flexible without the possibility of changes in length through deformation, observable only at its extremities B and C. Inverting the catenary on the ED axis will create an arch of ideal proportions. Poleni also quotes Gregory, who before Stirling, establishes the catenary as the ideal shape for the arches, "*Solae Catenariae sunt Fornices, sive Arcus legitimi*" (Gregory (1698) according to Riflessioni 1743, p.47, art.48).

Nevertheless, that what is of greater importance here are the assertions by Poleni himself as having put into practice the construction of catenaries, which, it appears, might have already been used to give form to vaulted structures, according to its own particular characteristics. He does not, however, elaborate any further on this particular point.

> As a result, it would also be possible to demonstrate how in practice catenary curves, since they are so easy to physically build, can serve (about which I, some time ago, drew up a scale model plan) to create a robust resistance to the arches and vaults, according to the particular characteristics of their structures. However, now is not the time to elaborate on such practices.
>
> (Riflessioni 1743, p.47, art.48)

Following the abstraction of the wedges as spheres, Poleni once again considers pressures exerted by these same spheres, Fig.7 (fig.7), and which accumulate between one another, beginning from those above. Poleni break these pressures down into two components, one horizontal and the other "perpendicular" (Riflessioni 1743, p.48, art.49). He says that Couplet used this breakdown of the forces and surmises that the three mathematicians did the same.

Despite the ill-defined nature of the value and breakdown of the thrusts, which, according to him "counter one another, in such a way that one cancels out the other, not so much in terms of lateral pressures but rather perpendicular ones" (Riflessioni 1743, p.48, art.49), Poleni realizes that the thrusts *at* or *ce* of Fig.7 (fig.7) get closer to the perpendicular at the base, that is, that "the lower the constituent parts of the arc are situated, the less they push outwards" (Riflessioni 1743, p.49, art.51). In this way, he reflects on the better performance of the sharp edges, that is to say, the gothic arch as opposed to that of the semi-circular type, especially when these have to support a significant weight in *C* or *e* such as the lanterns in the domes, Fig.10 (fig.8). The greater resistance provided by gothic (pointed) arches, is according to Poleni, the reason why they were so frequently used, for example, in the construction of the *Salon della Ragione* in Padova, in 1306, or in the dome of Santa Maria del Fiore in Florence, "the shape generated by the revolution in the Gothic Arch (named the Quarto Acuto) around its axis" (Riflessioni 1743, p.50, art.53). He also cites a passage in which Brunelleschi justifies the use of this pointed form (Riflessioni 1743, pp.50-1, art.53. He quotes Vasari).

The shape of the dome of St. Peter's in Rome is thus also pointed, as observed by Poleni in the corresponding section in the *Parere* and other books, it being extremely important for him that these plans accurately reflect the inner concave surface of the dome. With respect to the damage, Poleni demonstrates in this first manuscript that the dome's shape is more conducive to stability.

As a conclusion, Poleni attributes the damage to defective building material and building methods, and not to a potentially unstable structure. The vertical fissures of the segments in between the ribbing, he attributes to the yielding of the drum resting on the main arches due to the immense weight that the latter have to bear when compared to the piers. However, he does not attribute any greater importance to them since the ribbing, "the essential elements of the vault" (Riflessioni 1743, p.56, art.59), was found in near perfect condition except for a few superficial cracks. In fact, what especially confirmed Poleni in his opinion that the damage was not serious was the absence of horizontal fissures. The movement of the old tension rings may have been caused by anything, variation in temperature, earthquakes etc and could not be considered as evidence of the dome's condition, nor of it having moved significantly.

REMEDIAL ACTION

Poleni suggests two types of intervention in accordance with the recommendations of the three mathematicians and Sante Santini: restore as far as possible the structure to its original state and place new tension rings, despite the fact that the structure was not at risk.

The reparation of the fissures, considered necessary especially in the case of the main arches, could be conducted in various ways: placing bronze wedges into the cracks, lightly filling the inner surface with the purpose of inserting specially cut stones, finally filling the same cracks with a fine plaster. In this way not only would the visual aspect of the building be restored but also the parts comprising it, "which joined together in a certain way, would restore the equilibrium of the initial dynamics" (Riflessioni 1743, p.61, art.64).

In terms of damage of the drum, and repairing it, iron rings were considered as a useful solution. These would reinforce the cylindrical structure of the drum in the same way as the ties of an arch. However, he did not justify the size of the tension rings with respect to the overall stability, but rather in terms of their length which would vary according to their position.

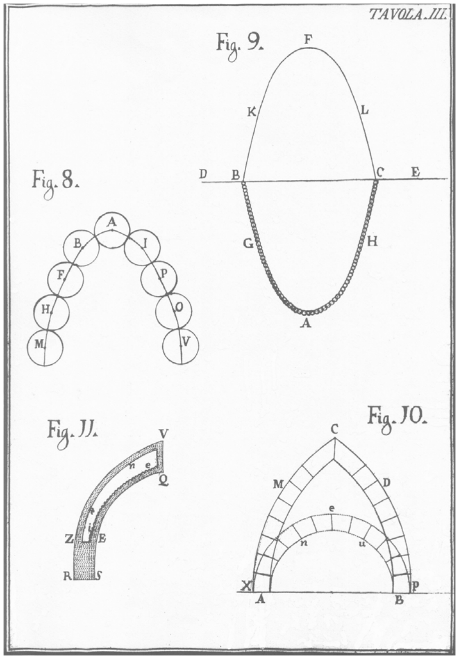


Figure 8. The principle of inverted catenary (Riflessioni 1743, pl. III)

The first one (1 palm x 3 ½ ounces) would have to be placed below the base of the buttresses, above the vault of the passage in *ee,* Fig.6 (fig.7), and the second one, at the outside base of the attic, the section of which could be reduced in height by 2 ounces by being shorter and situated higher up: "This would prevent pressure from being exerted against the buttresses" (Riflessioni 1743, p.65, art.68).

Afterwards, in the area of the shell, it would be necessary to monitor the state of the old rings. Furthermore, it seemed like a good idea to place a new ring (8 x 3 ounces) immediately above the lower windows which let light into the interstitial space between the vaults, "a place particularly suitable in relation to the outer thrusts of the dome" (Riflessioni 1743, p.67, art.70), and the other in the upper part of the dome, (N in fig.2), which only indicates that the dimensions would be smaller, also because of a narrower diameter.

It is also of interest to note several recommendations by Poleni, who at all times respects the final decision of the architects in charge of the restorations, regarding the exact location of the tension rings and their details. He insists that the quality of the metal used and the workmanship in shaping them should be of the highest possible standard, and recommends joining several sheets of iron together instead of using one single thick sheet to avoid hollowness. On the other hand, he suggests that the rings should be placed in outer part of the section because of the faulty masonry.

Lastly, among those interventions which Poleni held in doubt was to anchor the buttresses to the drum by means of bars, Fig.6 (fig.2) and (fig.6). As for the remaining measures, none seemed suitable to him: neither the remaining ironwork, apart from the tension ring, since these would increase both the weight and the cost, nor the enhanced thickening of the buttresses, or the placing of a type of support with statues in its upper section, according to Michelangelo's initial design, "since the highest sections of the buttresses exert the least resistance" (Riflessioni 1743, p.70, art.76). Nor did he believe that the spiral staircases should be filled in, nor the number of archways reduced, or the raising of four large supports above the piers to counter the dome's weight, which would only serve to increase the weight on the former. Still less was there a question of bringing down the lantern or substituting the lead covering for copper: "The heavy slabs, because of their weight must be left in place as the most suitable means of resisting outward thrusts on the structure" (Riflessioni 1743, p.73, art.80).

According to Poleni, the dome of St. Peter's was in no danger of collapsing, yet it was fitting to support any further damage. However ingenious, the model put forward by the *Parere* was erroneous. The origin of the damage was to be attributed to the effect of the weight on faulty construction. As for the physical intervention it was considered appropriate to insert the four iron rings and restore the main arches and the structure of the dome, filling in the fissures and restoring it to its original appearance.

> A subsequent investigation and the act of closely scrutinising all defects, taking stock of and identifying each and every detail, drawing up an historical description, would be considered useful for future reference and an aid to those who, in distant places, might be required to study the constitution of this structure.
>
> (Riflessioni 1743, p.75, art.82)

POLENI'S TRIP TO ROME

Shortly after sending the "Riflessioni", Poleni received a favourable reply from Benedict XIV, who deemed it necessary for him to travel to Rome in order to carry out a more thorough inspection of the dome. His stay in Rome was extended from May 1st until 19th June.

The meticulous work carried out by Poleni in Rome would in essence be that of observing and gathering information as to the state of the dome, while at the same time studying the never ending flow of reports. He even spoke of a model of the dome which Cosatti had presented to him as a gift. On his departure to Padova, Poleni handed over to the Pope the "Stato de' Difetti", which included the data and drawings made during his 18 visits with the architect Vanvitelli, and of a new manuscript where he made his final recommendations, "Aggiunta alle riflessioni", and all the reports which had been provided for him to examine.

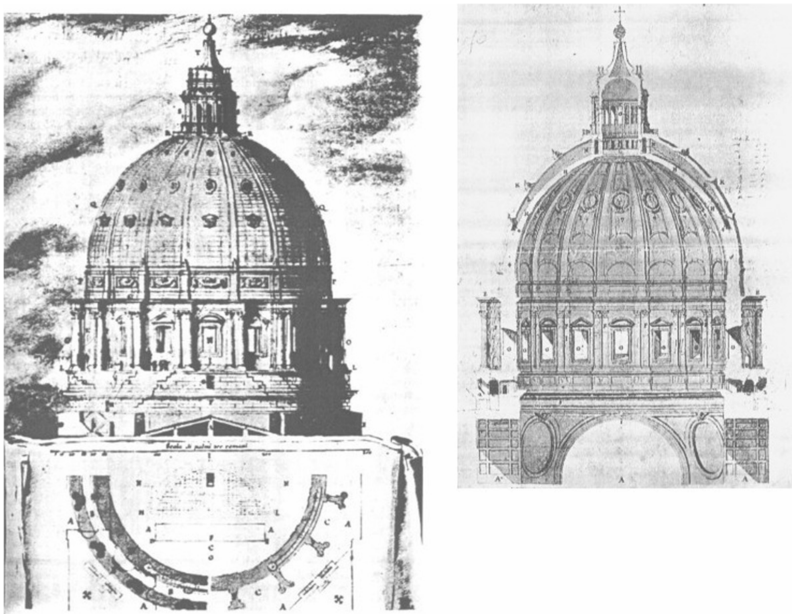


Figure 9. Elevation and section of the cracked dome of Saint Peter's by Vanvitelli (Di Stefano 1973)

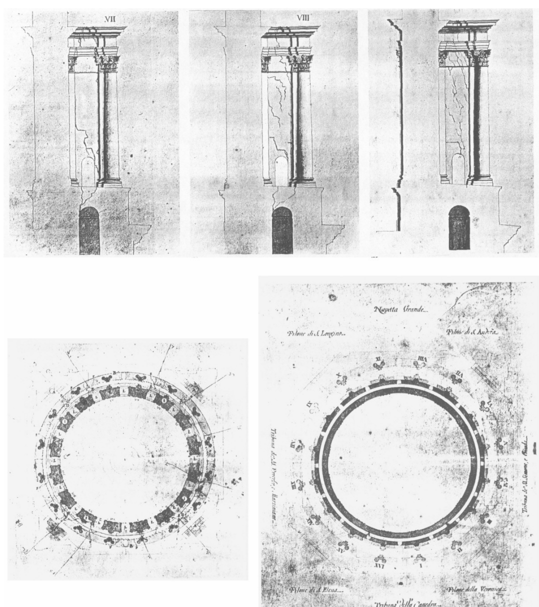


Figure 10. Cracked counterforts and plans of the dome of Saint Peter's by Vanvitelli (Di Stefano 1973)

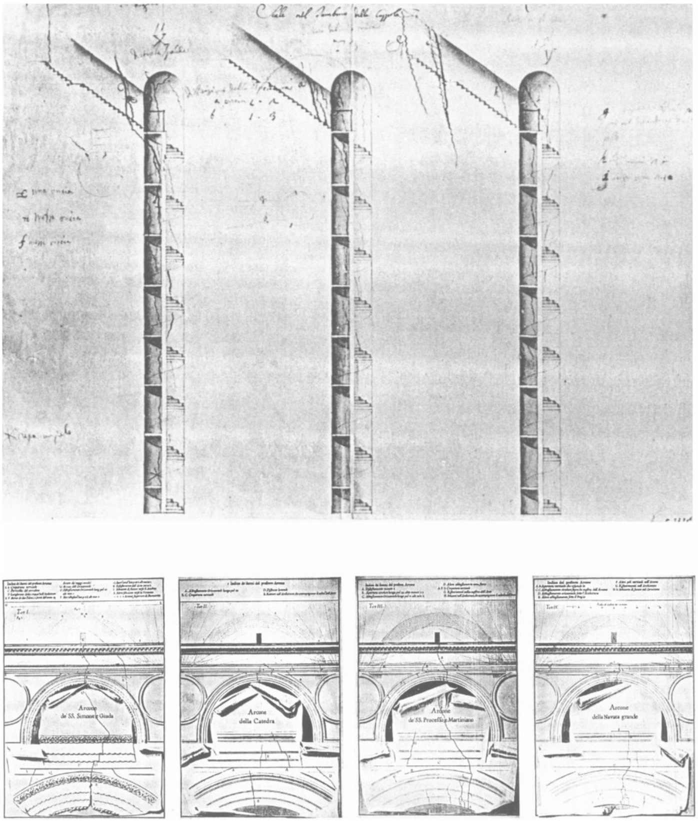


Figure 11. Cracked spiral staircases and main arches of the dome of Saint Peter's by Vanvitelli (Di Stefano 1973)

"AGGIUNTA ALLE RIFLESSIONI"

After visiting the building, Poleni observed that the piers were intact, the cracks in the main arches were few and insignificant and the tilting of the buttresses was less than had been supposed. Nevertheless, in some cases the cracks ran across the masonry, certain marble wedges put to measure the cracks advance were broken and some cracks ran across the inner parts of the lantern. In particular, most of the cracks which appeared in the drum had occurred in the location of the spiral staircase, "leaving the four major intermediate portions of the drum imperfectly joined to one another" (Aggiunta 1743, p.2, art.4). All this served to convince him that although the damage was not dangerous, it could in time, deteriorate the dome. All unanimously agreed that it was expedient to use iron rings to solve the problem, despite disagreement on how the problem had originated.

Nevertheless, there are some variations regarding the proposals contained in "Riflessioni". In the main arches there was talk once again of driving wedges into the cracks, either using bricks cut to size or pieces of marble. Then, however, it was thought best to insert iron wedges specially shaped to fit the dimensions of each crack of the main arches and dome. In his first manuscript he had considered bronze more suitable in order to avoid the problem of rust.

With respect to the tension rings it was decided to standardize the dimensions of the sections which were now to be 5 x 3 ounces, or even allowing for a small margin of error which could occur when working with large pieces of iron. Their location also varied slightly, especially those which were to be placed at a higher level (fig. 9 would be used to indicate the location of the iron rings). The first ring had to be placed at the base immediately below the cornice FF, the second in the lower part of the attic, KK. Those belonging to the upper sections were now to be placed at a point slightly lower down, in the MM in the springing of the dome and underneath the second level of small windows in the outer shell (fig. 13).

Not indicated in the "Riflessioni" was the method used for installing the tension rings. Previous methods had apparently involved using sixteen pieces of iron joined together. However, Poleni now prescribed the use of twenty-three or twenty-four iron bars, particularly at the higher levels, which were supposed to cover the outer projections of the ribs. Otherwise it would be necessary to "make large perforations in order to run the rings through" (Aggiunta 1743, p.6, art.12) Furthermore it was necessary to embed these in the walls, in such a way that the structure itself would prevent any movement of the tension rings, plaster them with lime to prevent rust and test the resistance of the joints. Summer was chosen in preference to winter for carrying out this part of the work.

Finally, he quoted a piece of writing signed by several experts. Theodoli, Ghezzi, Salvi, Hostini and Vanvitelli, in which tension rings were also suggested as an immediate measure. Once and for all, with these measures in place damage would no longer re-appear, or at least, they would ensure sufficient time during which to study the problem in greater depth.

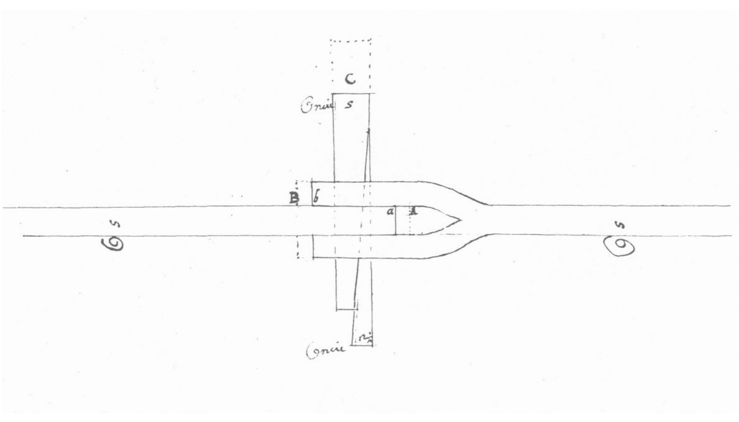


Figure 12. Detail of the joints between the bars of the iron rings to be placed around the dome of Saint Peter's (Vanvitelli 1743)

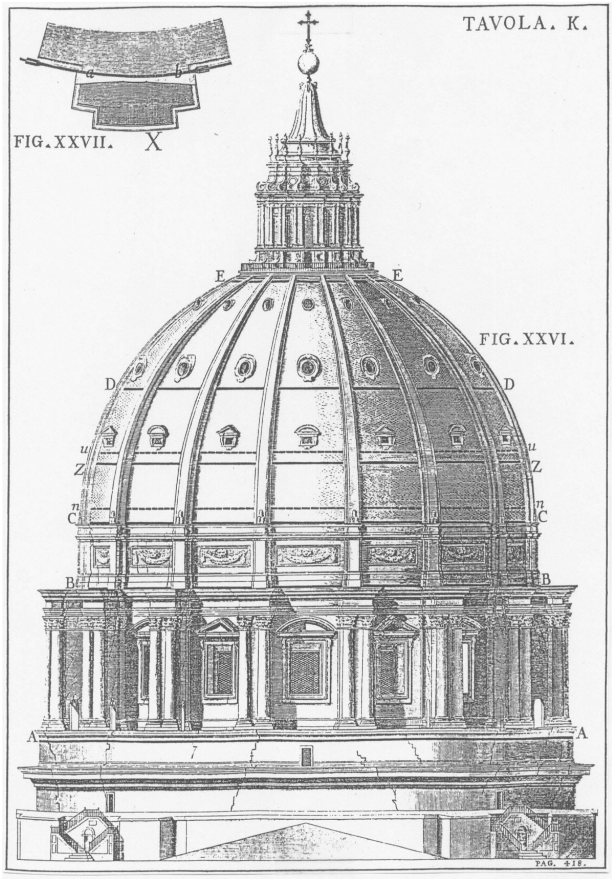
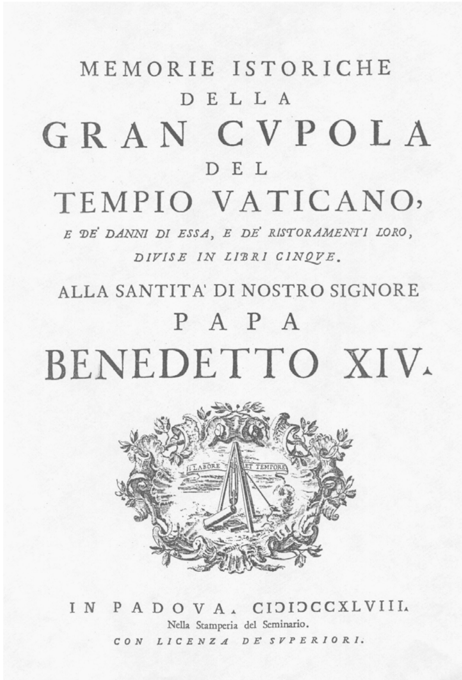


Figure 13. Location of the six new iron rings placed around the dome of Saint Peter's (*Memorie* 1748)

CONCLUSIONS

It is well known that Poleni´s analysis of the dome based on the catenary principle, included in his *Memorie* (1748), was the first time that the safe theorem of the Limit Analysis was applied to a masonry structure. However, his manuscripts are not so well known as the *Memorie* but they are very interesting because Poleni solved the problem of the dome in these documents. No quantitative structural analysis was included in the manuscripts, but when he left Rome, he gave Vanvitelli, main architect of the basilica, orders about the number, location and dimensions of the iron rings to put around it during the summer of 1743. His *Memorie* served as a record of the process of study and restoration of the dome though included his detailed analysis of the stability of the dome, but the new rings had been around the dome since 1743. In his manuscripts Poleni wrote about the theory of vaults that was developing at that time but it seems another case where practice prevailed over the theory.


MEMORIE ISTORICHE
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E DE' DANNI DI ESSA, E DE' RISTORAMENTI LORO,
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ALLA SANTITA' DI NOSTRO SIGNORE
PAPA
BENEDETTO XIV.

IN PADOVA. CIƆIƆCCXLVIII.
Nella Stamperia del Seminario.
CON LICENZA DE' SVPERIORI.

Figure 14. Frontispiece of Poleni´s *Memorie* (1748)

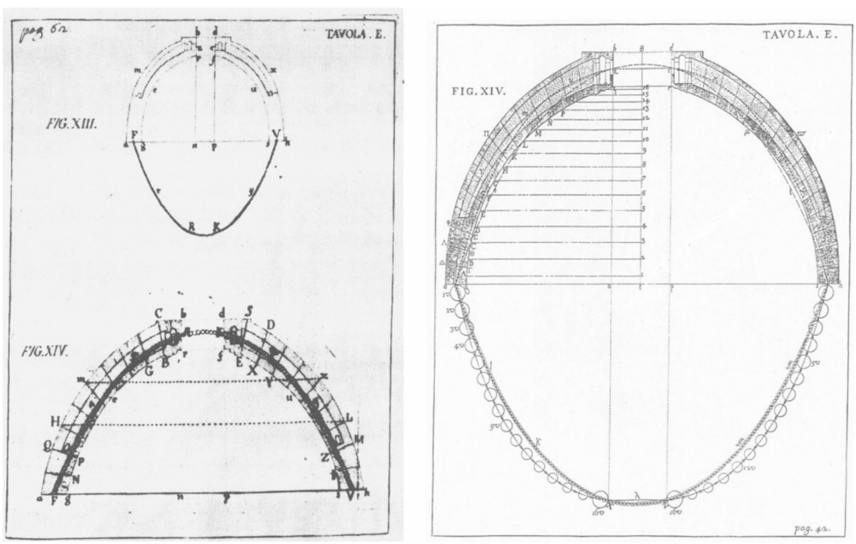

Figure 15. On the left, preliminary sketch of plate E by Poleni (1748a, f. 234) (Brusatin 1971) and on the right, analysis of the stability of the dome of Saint Peter's (*Memorie* 1748)

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