On the Relationship between Construction Engineering and Strength of Materials in Gerstner's "Handbook of Mechanics"

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INTRODUCTION

During the cannonade of Valmy on 20 September 1792, the French revolutionary troops for the first time withstood the pressure from the counter-revolutionary alliance formed by Austria, Prussia and several small German states. During the First War of the Coalition, the revolutionary troops switched from the defensive to the offensive. By the end of the war in 1795 they had conquered Savoy, all territories to the left of the River Rhine, and the Netherlands. In view of the important role of higher technical education for the victory of the French revolutionary army, a *Hofkommission für Revision des öffentlichen Schulwesens* (Royal commission for the revision of the public school system) was established in Austria in 1795. Franz Joseph Ritter von Gerstner (1756-1832) (**fig. 1**), professor for higher mathematics at Prague University, became first among equals of the commission.

Figure 1. Franz Joseph Ritter von Gerstner (1756-1832) (Gerstner 1833)

On 27 September 1797, Gerstner's ideas concerning a reform of natural science studies even met with approval from Emperor Francis II: "During the reorganisation of philosophical studies I also expect a detailed plan for a higher technical institute to be developed, the benefit of which should already be apparent from any drawing being submitted" (Kraus 2004, pp. 125-6). During the following year Gerstner submitted such a "detailed plan", which was received enthusiastically by the Bohemian Estates. After several modifications of Gerstner's original plan due to concessions resulting from the uncertainties surrounding the Napoleonic Wars, the *Königliche technische Ständelehranstalt* (today know as Prague Technical University) opened on 10 November 1806 as the second oldest technical university in Europe. Gerstner was head of the "Polytechnikum" until 1822, and also head of the mathematics and mechanics departments. In 1811 the emperor made him water engineering director for Bohemia. Franz Joseph Ritter von Gerstner became a sought-after adviser for numerous technical projects in his homeland. Together with his son Franz Anton Ritter von Gerstner (1793-1840) (**fig. 2**), he was responsible for the construction in the 1820s of the first railway line in Continental Europe, i.e. the horse-drawn railway between Linz and Budweis (Kurrer 1990b, pp. 5-6).

Figure 2. Franz Anton Ritter von Gerstner (1793-1840) (Photo: Picture archives of Austrian national library)

The three-volume *Handbook of Mechanics* published by Franz Anton Ritter von Gerstner (1831-1834), based on his father's lectures (**fig. 3**), was the first comprehensive German-language book on applied mechanics. The subtitle says: "[...] extended with recent English structures [...] by Franz Anton Ritter von Gerstner" (s. **fig. 3**). The list of advance orders published in 1834 reads like a *Who's Who* of contemporary technology in Continental Europe. Of approx. 1900 advance orders, 30 % came from the construction and transportation sector, 23 % from the book trade, 10 % from the trade, factory, machine and iron and steel sector, 7 % from libraries, 5 % from the mining industry, 4 % from research and education, 4 % from the military, 3 % from agriculture and forestry, and 14 % from other sectors (Kurrer 1990a, p. 502). It was the culmination of Franz Joseph Ritter von Gerstner's technical/scientific lifework.

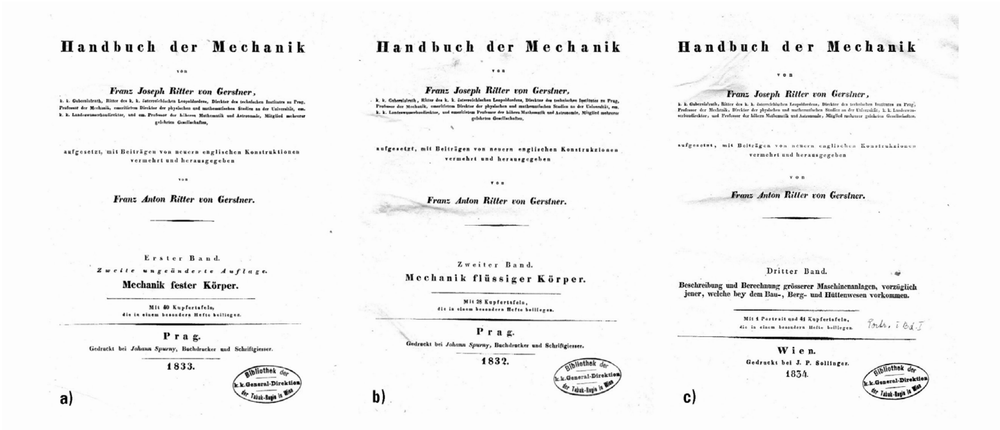

Figure 3. Title pages of a) first, b) second and c) third volume of the *Handbook of Mechanics* (Gerstner 1832-34)

The *Handbook of Mechanics* had significant impact on the constitution phase of the techno-scientific fundamental discipline of applied mechanics in the German-language countries between 1830 and 1850, alongside the classic French contributions from scientists such as Navier, Poncelet, Coriolis etc., which did, however, not emerge until the 1840s. What's more: the Gerstner publication under the modest title of *Handbook of Mechanics* was **the** preeminent German-language compendium of science-based technology in the first half of the century. During the transition phase between small-scale manufacture and the technology of the industrial revolution, the Handbook elaborated on contemporary construction and machine engineering in such a scientific way that, from today's point of view, it can be regarded as a turning point in the establishment of the system of classic engineering sciences.

GERSTNER'S THEORY OF MATERIAL STRENGTH

Gerstner introduced the distinction between applied mechanics and theoretical mechanics (**fig. 4**). Gerstner was the first German-speaking engineering scientist to define applied mechanics as a subject, and he introduced the term "Technische Mechanik" into the German language (Kurrer 1990a, p. 503):

> Applied mechanics (mécanique industrielle ou mécanique appliquée aux arts) deals with any process relating to the manufacture of trade or artistic products and their representation according to the necessities to be satisfied. All human work is partly carried out manually, and partly using tools and machines. For both types of work, training in mechanics should convey the laws and rules according to which workers should conduct themselves and prepare, check and use their tools and machines properly. For any type of work a force is required (...). When it comes to mechanics, we therefore

only have to study the laws of nature carefully in order to examine the properties of the mechanical bodies we want to process, which will enable us to determine the most appropriate means for achieving the desired result.

(Gerstner 1833, p. 3)

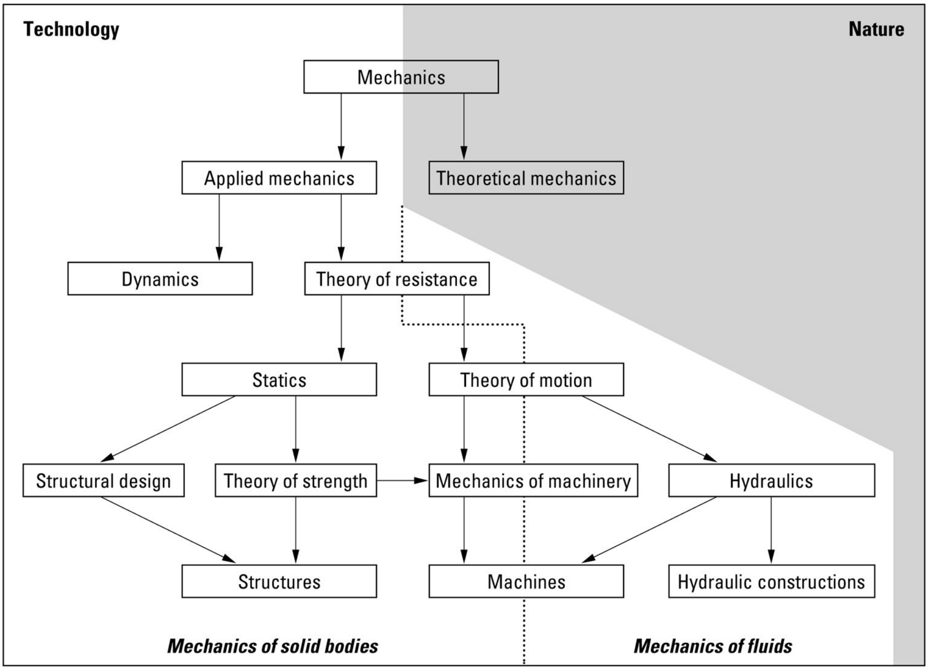

Figure 4. Structure of mechanics according to Gerstner (diagram by author)

Gerstner's work essentially followed the concepts established by Jean-Victor Poncelet (1788-1867), whose fundamental contributions significantly influenced the constitution phase of applied mechanics (1825-1850). Poncelet's lectures and seminars on *mécanique industrielle* at *École d'Application de l'Artillerie et du Génie* in Metz were documented – like others, too – by the in-house lithographic unit with print runs of about 500 copies (Kahlow 1994, p. 75). Poncelet researcher Konstantinos Chatiz, for example, refers to three lithographic Poncelet lecture manuscripts on *mécanique industrielle* edited by M. Goesselin between 1827 and 1830 (Chatzis 2002, p. 353).

For applied mechanics Gerstner distinguished between dynamics and "resistance theory, which examines the magnitude of the resistances and the laws according to which they counteract the respective forces" (Gerstner 1833, p. 6). Gerstner subdivided *resistance theory* further into theory of frictional resistances, structural design and theory of material strength. Gerstner still treated structural design and theory of material strength, which were later consolidated into structural theory, as separate entities. While Gerstner regarded structural design as the application of structural analysis in the sense of balance theory to the field of construction engineering, he regarded the theory of material strength as a wider field:

> In machine and building construction it is important to make all components strong and thick enough in order to prevent them from tearing, excessive bending or even breaking. On the other hand the components should not be excessively strong and heavy, in order to avoid unnecessary costs or resistance against movement, or indeed breaking under their own weight.
>
> (Gerstner 1833, p. 241)

Based on common practice at the time, Gerstner distinguished between four load types:

- Absolute strength (tensile strength)
- Relative strength (bending strength)
- Reactive strength (resistance to compression)
- Resistance against turning (torsional strength)

Like no other before him, Gerstner based the theory of material strength on experiments. With his empirically accentuated theory of material strength Gerstner established the basic principles of a physically substantiated theory of proportions for structural and mechanical engineers, which formed the historical/logical transition to structural theory and classic dimensioning theory.

GERSTNER'S EXPERIMENTAL STRESS ANALYSES

Gerstner's experiments on the tensile strength of wrought iron and steel, for example, set standards in terms of precision, methodology and application. As part of the design work for a chain suspension bridge in Prague he developed a polynomial formula for the stress-strain diagram of steel and wrought iron, the parameters for which he determined himself through experiments. Gerstner was the first to clearly differentiate between a linear region ("perfect elasticity") and a nonlinear region ("imperfect elasticity") and warned against exceedance of the proportionality limit, particularly for chain suspension bridges. He also dealt extensively with the bending strength of timber and iron structures. Unlike Navier (Navier 1826), Gerstner did not succeed in formulating an inherently consistent technical bending theory.

Tensile strength of iron

Gerstner's intensive studies on the strength of iron were based on the application of iron in the construction of chain and cable suspension bridges. In 1824, Gerstner was commissioned – in his capacity as water engineering director for Bohemia – to assess the suitability of Bohemian iron for the construction of a chain suspension bridge across the Vltava river in Prague (Gerstner 1833, p. 259). By then, suspension bridges had already been built in several countries, for example

- in the United States of North America, based on the bridge system patented by James Finley in 1801 (Peters 1987, pp. 28-33);
- in Switzerland since 1822, based on designs by Guillaume Henri Dufours (Peters 1987, pp. 79ff.);
- in France since 1823/24, based on designs by Marc Seguin since 1822 and Navier (Wagner and Egermann 1987, p. 68);
- in Russia since 1823/24, based on designs by William Traitteur (Fedorov 2000, pp. 123-51);
- in Austria since 1824/25, based on designs by Ignaz Edler von Mitis and Johann von Kudriaffsky (Pauser 2005, pp. 92-3 and 123-25);
- and in Bohemia-Moravia since 1823/24, based on designs by Friedrich Schnirch (Hruban 1982).

The suspension bridge across the Menai Strait in Wales with a span of 175 m was built between 1818 and 1826, based on designs by Thomas Telford. Telford's suspension bridge became and remained the key chain bridge structure until well beyond the middle of the 19th century. What was the situation regarding suspension bridge construction in Germany at the time? For the time being, several proposed projects failed to come to fruition: The young Johann August Röbling had been working on suspension bridges since 1824/25, but had failed with three designs by the end of 1828 (Güntheroth and Kahlow 2005, pp. 127-8).

The Prague bridge project encouraged Gerstner to press on with his largely experimental theory of material strength. Gerstner introduced his chapter on *Absolute strength of iron* (Gerstner 1833, pp. 242-59) with the proportionality law for linear-elastic bodies, i.e. Hooke's law:

$$q : Q = [(f \times \alpha)/l] : [(F \times \alpha')/L] \qquad (1)$$

In Gerstner's words: When deforming two fully elastic bodies of different dimensions but identical material, weights q and Q behave like the products of their cross-sectional areas f and F in relation to expansion α and α' and length l and L (Gerstner 1833, p. 243). In his theory of material strength Gerstner keeps referring back to proportion (1). Gerstner's proportion (1) is followed by a discussion of the experiments undertaken by Musschenbroek, Eytelwein, Rennie, Telford und Brown, Brunel, Barlow, Tredgold, Navier, Dufour, Rondelet und Soufflot (Gerstner 1833, pp. 253-9). He also carried out extensive experiments himself (Gerstner 1833, pp. 259-63). In his plate catalogue, Gerstner presented a drawing of Eytelwein's experimental device for determining the tensile strength of iron (**fig. 5**), taken from the second volume of Eytelwein's *Handbook of structural analysis of solid bodies* (1808). While such experimental devices enabled the tensile strength of iron bars to be determined, it was not possible to determine their force-deformation behaviour before tensile failure. Gerstner went on to summarise the tensile strength experiments carried out by Musschenbroek, Rondelet and Soufflot, Navier, Dufour, and Telford and Brown in tabular form.

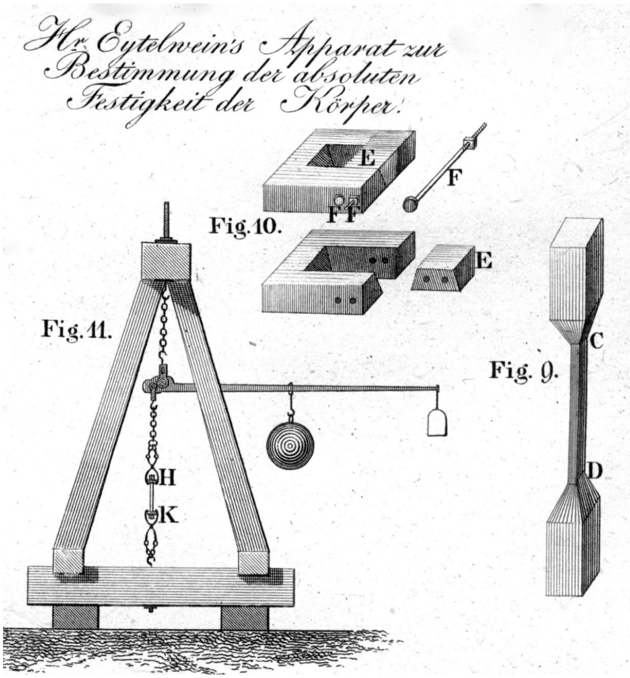

Figure 5. Eytelwein's device for determining tensile strength
(Gerstner 1832-34, plate no. 14, figs. 9-11)

Laws relating to the strength of iron

Gerstner used 13 series of experiments with piano wires of different diameter as the basis for his insights into the *laws relating to the strength of iron*, which went far beyond the state of knowledge of the authors he referred to. For his experiments Gerstner designed a special device (**fig. 6**), consisting of a 4.73 m long lever arm CA used as indicator for scale DA, a shorter lever arm BC with counterweight H, and a sliding weight. The test wire mn with a length of approx. 1.25 m is fixed at its lower end n, fed over a roller at its upper end m, and tensioned via a spindle and a toothed wheel. Indicator CA enlarges elongation Δl 54-fold and shows the value on scale AD. Using this experimental device, Gerstner succeeded in measuring and recording the force-deformation behaviour of iron wires before tensile failure algebraically with high precision. He also succeed in clearly separating the total elongation Δl_{test} into an elastic $\Delta l_{elast,test}$ and a plastic $\Delta l_{plast,test}$ component.

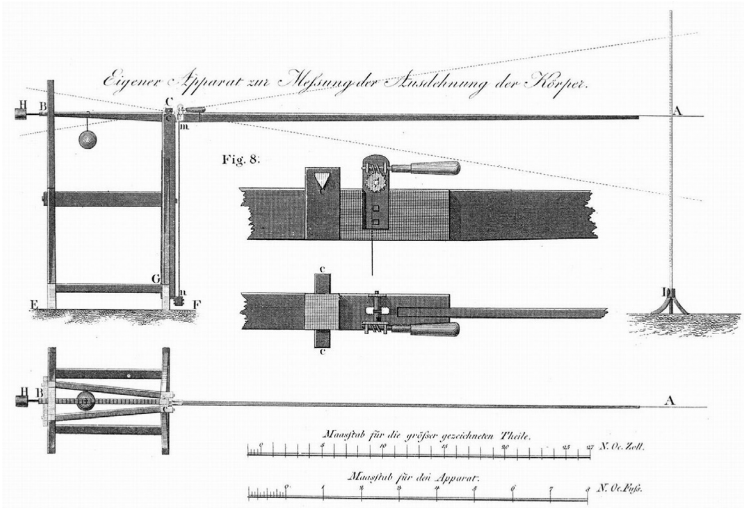

Figure 6. Gerstner's device for determining the force/deformation diagram of iron wires
(Gerstner 1832-34, plate no. 14, fig. 8)

Gerstner used a polynomial approach to derive the parabolic and non-dimensional force-deformation law for iron wires (Gerstner 1833, p. 265):

$$F(\Delta l)/F_{max} = (\Delta l/\Delta l_{max}) \times [2 - (\Delta l/\Delta l_{max})] \quad (2)$$

Eq. (2) is shown schematically in **fig. 7a**: "This expression contains the general equation describing the relationship between the greatest weight F_{max} a wire can carry, the maximum elongation Δl_{max} caused by this weight, a random load $F(\Delta l)$ and the associated elongation Δl" (Gerstner 1833, p. 265). Gerstner determined the parameters F_{max} and Δl_{max} based on 13 series of experiments. For small values of $\Delta l/ \Delta l_{max}$ eq. (2) becomes a linear relationship. For the purpose of verifying eq. (2), Gerstner used his experimental device for carrying out 10 series of experiments with piano wires, ordinary wires, steel mainsprings, and annealed wire (s. **fig. 6**). He found that any deviations between the experiments and the calculation were small.

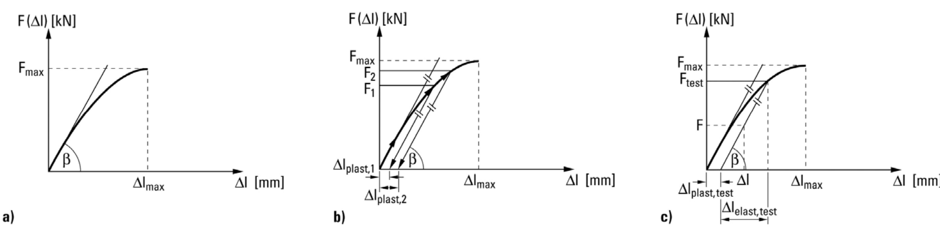

Figure 7. Schematic force/deformation diagrams for iron according to Gerstner (diagrams by author)

Gerstner went on to generalise his approach according to eq. (2): "The 10 experiments described above will suffice for establishing general laws on the strength, elongation and elasticity of bodies in general, and iron in particular" (Gerstner 1833, p. 272):

$$F(\Delta l) = \Delta l \times (a_1 - a_2 \times \Delta l) \qquad (3)$$

According to Gerstner, parameters a_1 and a_2 have to be determined experimentally for each material type. Finally, Gerstner examined "how closely the iron elongated by weight F(Δl) would return to its original length once the weight is removed" (Gerstner 1833, p. 272). Through analysis of the experimental result he found the following relationship (**fig. 7c**):

$$\Delta l_{test} = \Delta l_{elast,test} + \Delta l_{plast,test} \qquad (4)$$

The phenomenon of strain hardening of iron found by Gerstner is shown in **fig. 7b**: Load between 0 to F_1, relief along Hooke's line up to permanent elongation $\Delta l_{plast,1}$, load along the Hooke's line up to F_1 and then along the curve up to F_2, and finally relief along Hooke's line up to permanent elongation $\Delta l_{plast,2}$. Gerstner used his insights on strain hardening of iron for the purpose of dimensioning chain suspension bridges by suggesting that the chain bars should be pretensioned prior to installation up to a force F_{test} with adequate distance from F_{max} ($F_{test} = F_{max}/3$), and to only permit forces in practice that meet condition $F \leq F_{test}/2$. In this context Gerstner criticised the then common advice "that for achieving sufficient safety the iron bars should only be subjected to half the failure load" (Gerstner 1833, p. 277). In practice, this would lead to excessive deformation of suspension bridges.

The weakness of Gerstner's semi-empirical iron strength theory was that he only offered tables and algebraic formulas, but no force-deformation diagrams. Gerstner was therefore unable to detect the phenomenon of yield. "Drawing is the language of the engineer" were the concise and visionary words used by Karl Culmann (1821-1881), the inventor of graphical statics, in the foreword of his monograph *Die Graphische Statik*, published in 1866. Around 1830, Poncelet and his disciples at *École d'Application de l'Artillerie et du Génie* in Metz had already recognised the power of the drawing in the context of the techno-scientific cognitive process, as discovered by Andreas Kahlow in a report to Poncelet (Fond Poncelet, Carton 7, doc 149 "Note sur les Expériences à Metz pour étudier la résistance de l'extension dans le fils métalliques"), which contained a force-deformation diagram for iron that not only clearly showed Hooke's range and the yield plateau, but also the modulus of elasticity, albeit without specification of the dimension (**fig. 8**).

Based on his insights into the laws relating to the strength of iron, in the *structural design* chapter of his handbook Gerstner developed a viable dimensioning theory for suspension bridges derived from a critical analysis of chain suspension bridges built in England, France and Germany, which he used for designing a chain suspension bridge with a span of 142.25 m and a sag of 10.75 m across the

Vltava river in Prague (Gerstner 1833, pp. 449-88). A brief synopsis of Gerstner's theory of suspension bridges can be found in the monograph *Geschichte der Baustatik* (Kurrer 2002, pp. 109-11).

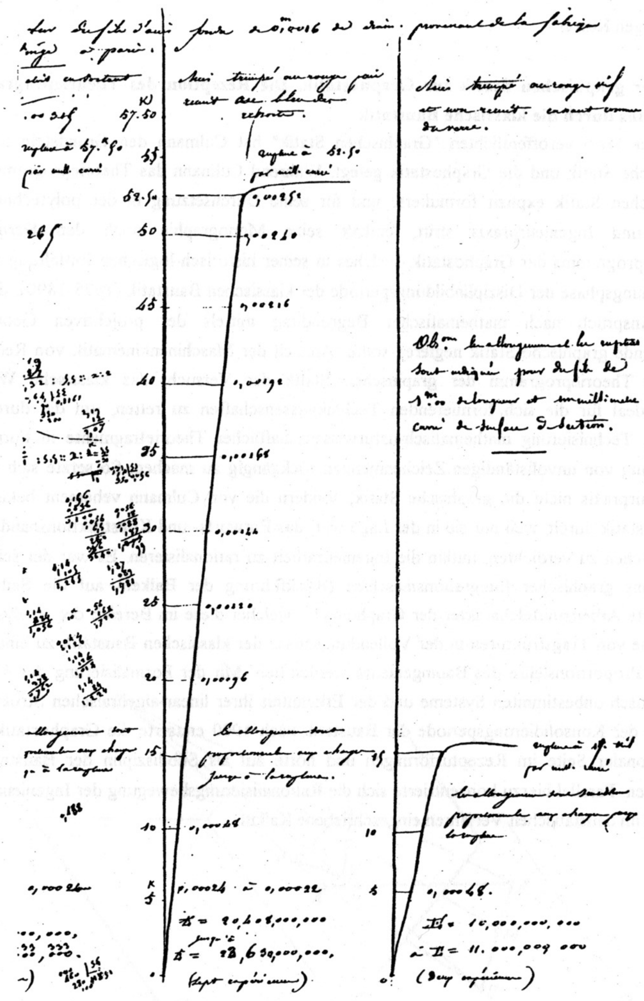

Figure 8. Force/deformation diagram for iron from a report to Poncelet c. 1832 (Kahlow 1994, p. 101)

Bending strength

Gerstner's bending theory is basically an empirical theory of proportions for structural and mechanical engineers. In his chapter on bending strength Gerstner initially analysed the bending fracture problem in the tradition of Galilei (Gerstner 1833, pp. 291-322), followed by an analysis of the functional state of bending beams (Gerstner 1833, pp. 322-64).

For a cantilever beam with length L, cross section width B and section height H subjected to point load E_U (bending failure load) at its end, he derived the following formula:

$$E_U = (m \times B \times H^2)/L \qquad (5)$$

The proportionality factor m in eq. (5) is material- and cross-section-dependent and has to be determined through experiments. If ideal elastic and ideal plastic material behaviour was assumed with σ_{yield} as yield stress, m would have the following values for building materials with pronounced yield point:

- In the elastic limit state: $m = \sigma_{yield}/6$
- In the fully plastic state (limit load or bending failure force): $m = \sigma_{yield}/4$

Gerstner collated a table for values of m based on an analysis of bending failure experiments for oak, elm and pine beams carried out by Musschenbroek (Gerstner 1833, p. 297). Gerstner derived further values for m for single-span beams with centrally applied bending failure force from experiments carried out by

- Barlow for fir timber,
- Eytelwein for various timber types,
- Rondelet for cast iron,
- Tredgold for different types of natural stone, and
- Barlow for brick.

Gerstner then went on to calculate the limit load of simple structural systems for point loads and equally distributed linear loads. According to Gerstner, one important application of the theory of material strength is dimensioning of bridges (Gerstner 1833, p. 311). He thus developed the structural similarity relationship for a simple wooden bridge model, consistently distinguishing between permanent load (own weight) and live load (e.g. horse-drawn carriages).

Gerstner's deductions on the bending strength of cantilever beams with rectangular, circular and elliptic cross section with point load E_U at the end were far-sighted (Gerstner 1833, pp. 316-7). They were based on the concept of stress (in the sense of failure stress). Gerstner calculated the restraint moment M for a rectangular section as

$$M = E_U \times L = (k/f) \times [(B \times H^2)/6] \qquad (6)$$

where k/f is the failure stress. For ideal elastic and ideal plastic material behaviour, k/f is the yield stress σ_{yield}. Comparison of the coefficients in eq. (5) and (6) results in $m = \sigma_{yield}/6$. The expression in the square bracket in eq. (6) is the section modulus of the rectangular section. Gerstner determined the stress k/f through experiments. In a similar way, for circular and elliptic cross sections he split the product into failure stress and section modulus using infinitesimal calculus. Although eq. (5) and eq. (6) resolved for E_U are mathematically equivalent, eq. (6) expresses the physical core of bending theory much more clearly than eq. (5). Unfortunately, Gerstner did not move beyond the theory of proportions for beam bending.

For the purpose of bending dimensioning, Gerstner specified safety coefficients of 10 for wood and 2 to 3 for iron, which concludes the section on Gerstner's analysis of the state of failure on bending.

Bending laws

With regard to the analysis of the functional state under bending stress, Gerstner's main concern is to limit deformation – in modern terms fitness for purpose:

> A general requirement is to protect machine or building components not only against failure, but also to ensure adequate strength against bending and vibration. We therefore also have to consider bending of bodies. We know that long bars or beams, notwithstanding the fact that they are able to support a particular load, may bend to such an extent as to restrict the function of the machine through moving components becoming loose, or through teeth and gears becoming separated, for example. The same applies if the end supports of a bridge are able to carry a particular load, but bend to such an extent as to not only spoil the shape of the arches, but also to make traffic across the bridge impossible.
>
> (Gerstner 1833, pp. 322-3)

Once again Gerstner is able to foray into the theory or proportionality, this time due to the dominion of Hooke's law. Using the example of two cantilever beams, he finds the deflection ratio of the cantilever beam ends U : u (**fig. 9**) as

$$U : u = (Q \times L^3)/(B \times H^3) : (q \times l^3)/(b \times h^3). \qquad (7)$$

For bending tests, Gerstner preferred simply supported beams with test loads G or g at midspan. He derived a relationship similar to that of proportion (7):

$$W : w = (G/B) \times (L/H)^3 : (g/b)/(l/h)^3 \qquad (8)$$

As in the derivation of eq. (7) and (8), the factor $[(F \times \alpha')/L]$ or $Q/[(F \times \alpha')/L]$ from proportion (1) always appears in similar calculation procedures relating to the functional state under tension, bending, pressure and torsion. It corresponds to the modulus of elasticity E, as shown below.

As in for his analysis of the laws for iron, Gerstner refers to plastic deformations as a result of the elastic limit being exceeded during bending. In order to be able to quantify the laws relating to bending before failure, Gerstner designed an experimental device for determining the deflection at midspan of simply supported timber beams (**fig. 10**). In analogy to his experiments with piano wires, Gerstner carried out 14 series of experiments with oak bars and found "that, like the elongation of iron, the deflection initially remained proportional to the load, but later became larger" (Gerstner 1833, p. 329). For the relationship between test load F and deflection f at midspan he therefore used the same approach as in eq. (2) (see **fig. 7**):

$$F(f) = f \times (A - B \times f) \qquad (9)$$

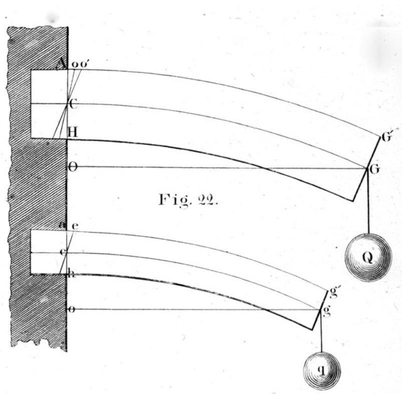

Figure 9. Bending beam according to Gerstner (Gerstner 1832-34, plate no. 15, fig. 22)

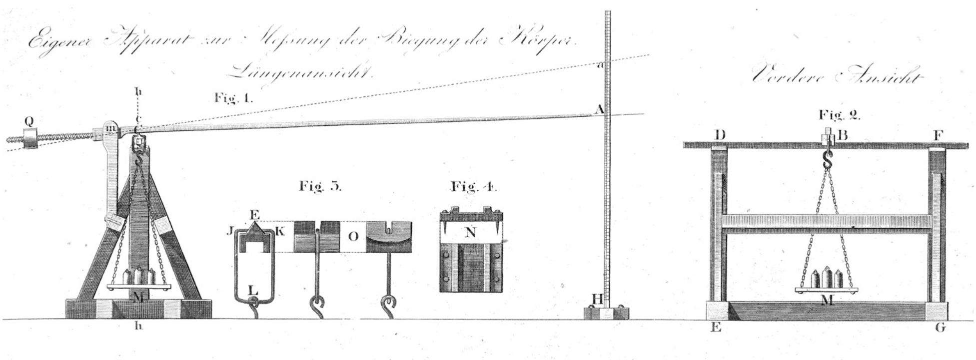

Figure 10. Gerstner's device for bending tests with wooden bars (Gerstner 1832-34, plate no. 16, figs. 1-4)

He determined parameters A and B from bending tests and concluded "that bending of bodies follows the same laws as elongation" (Gerstner 1833, p. 331). Comparison of calculated and experimental values shows little difference and reveals that, in the range relevant in practice, F(f) and f are proportional:

$$F(f) = f \times A \qquad (10)$$

The inhomogeneous structure of the wooden bars and ordinary iron bars "only very imperfectly follows the laws [...] found in the most perfect piano wires and mainsprings" (Gerstner 1833, p. 333). For this reason, Gerstner recommended that dimensioning for bending should be based on eq. (10), and the elastic limit should not be exceeded in practice. Using proportion (8) and bending tests according to (10), Gerstner was now able to numerically determine deflection W of wooden bars with different geometry.

Gerstner went on to describe further bending tests at Prague "Polytechnikum" for determining parameters A and B according to eq. (9):

- 11 bending tests with oak, beech, spruce and fir timber, with different fibre direction and number of annual growth rings (Gerstner 1833, pp. 338-44);
- 7 bending tests with cast iron bars from Bohemian iron works (Gerstner 1833, pp. 347-52);
- 7 bending tests with wrought iron bars from Bohemian iron works (Gerstner 1833, pp. 353-8).

Gerstner summarised the results as follows: "Based on the experiments described above and the equations derived from them, all bending dimensioning tasks for beams, rods or bars can be solved" (Gerstner 1833, p. 359). He thus considered the problem of beam bending to be solved.

Gerstner provided several examples for the application of beam bending theory. One of them was a comparison of the price/performance ratio of simply supported beams with identical geometry but different materials. The relationship between the central loads for beams with identical deflection at midspan made from oak, beech, spruce and fir timber, cast iron and wrought iron is as follows: 1:1.08:1.62:1.46:8.97:17.42. Gerstner calculated these values without taking into account own weight (Gerstner 1833, p. 358). Based on the material price at Welsh ports, Gerstner concluded that cast iron would be preferable in Wales, while timber would be preferable in Bohemia.

Compressive and buckling strength

According to Gerstner, the force-deformation behaviour of pressure-resistant materials is based on Hooke's law, just like tension-resistant materials. A modern notation of proportion (1) would be as follows:

$$F_1 : F_2 = (A_1 \times \Delta l_1/l_1) : (A_2 \times \Delta l_2/l_2) \qquad (11)$$

In proportion (11) F_1 and F_2 represent the forces, A_1 and A_2 the cross-sectional areas, Δl_1 and Δl_2 the elongation or shortening, and l_1 and l_2 the lengths of bars 1 and 2. Using stresses $\sigma_1 = F_1/A_1$, $\sigma_2 = F_2/A_2$ and elongation $\varepsilon_1 = \Delta l_1/l_1$ and $\varepsilon_2 = \Delta l_2/l_2$ in proportion (11) results in:

$$\sigma_1 : \varepsilon_1 = \sigma_2 : \varepsilon_2 \qquad (12)$$

Proportion (12) is in fact the modulus of elasticity E. As for tension, Gerstner chose the approach according to eq. (3) for the force-deformation diagram, with the difference that the second term is positive, and only the amounts should be used in the equation. Unfortunately, Gerstner was unable to determine the two parameters a_1 and a_2, since suitable devices for distortion measurements via compressive tests were not available at the time – in fact they were not developed until about 50 years later by Johann Bauschinger (Kurrer 2002, p. 146). He nevertheless realised that the force-deformation diagrams for tension and compression were not point-symmetric (Gerstner 1833, p. 366).

For the buckling strength of supports, Gerstner derived the critical load P_{crit} for the simply supported case (second Euler case). For a support of length l made from fir timber with rectangular cross section with side lengths b and h, the critical load takes the following form:

$$P_{crit} = [(Q \times L)/(B \times H \times \alpha')] \times [(b \times h^3)/12] \times (\pi/l)^2 \qquad (13)$$

Gerstner determined the expression $[(Q \times L)/(B \times H \times \alpha')]$ in eq. (13), which also appears in proportion (1), from bending tests with fir bars with rectangular cross section and side lengths B and H. Q represents the force, α' the elongation, and L the length of the test specimen. If cross-sectional area F = B × H, stress σ = Q/F, and elongation ε = α'/L, then

$$[(Q \times L)/(B \times H \times \alpha')] =[(Q \times L)/(F \times \alpha')] = \sigma/\varepsilon = E \qquad (14)$$

Gerstner referred to the expression in eq. (14) as the "elastic force ratio", Tredgold called it "modulus of elasticity" (Gerstner 1833, p. 384). With moment of inertia $I = (b \times h^3)/12$ and eq. (14), eq. (13) can now be written as follows:

$$P_{crit} = E \times I \times (\pi/l)^2 \qquad (15)$$

As an application of his insights into the resistance to compression, Gerstner sets himself the task of dimensioning the walls of a multi-storey building with increasing thickness towards the bottom, based on the basic principle "that all sections of the walls should be equally safe against crushing" (Gerstner 1833, p. 371). He concludes "that the thicknesses of the walls should increase from top to bottom in a geometric progression, if the number of floors increased according to the arithmetic series 0, 1, 2, 3, 4...n" (Gerstner 1833, p. 371).

Torsional strength

Gerstner described an experimental device he developed for determining the functional relationship between the torsional moment M_t and the angle of rotation φ, although he only examined solid circular cross-sections, hollow circular cross-sections, solid square cross-sections, and hollow square cross-sections. Gerstner applied his insights predominantly to dimensioning of machine elements such as shafts for water wheels, for example. A universal dimensioning technique for elastic torsion applicable to engineering practice was only developed in 1917 by August Föppl (Kurrer 2002, pp. 288-9).

GERSTNER'S SYNOPSIS OF THE THEORY OF MATERIAL STRENGTH

Gerstner summarised his theory of material strength on two printed pages. He pointed out that the factor $[(Q \times L)/(F \times \alpha')]$ appears in the load types of tension, bending, buckling and torsion. However, according to eq. (14) this factor is identical to the modulus of elasticity E. The task of the theory of material strength therefore is to:

- select three parameters from a total of four from the proportions for tension and calculate the fourth one;
- select four parameters from a total of five from the proportions for bending and calculate the fifth one;
- select three parameters from a total of four from the proportions for buckling and calculate the fourth one;
- select four parameters from a total of five from the proportions for torsion and calculate the fifth one.

For the case of bending of a beam with two supports, length l, rectangular section with b (= width) and h (= length) and central point load G, Gerstner specified the following proportion (Gerstner 1833, p. 326):

$$G : (b \times h) \times [(4 \times h^2)/l^2] \times (U/l) = Q : [(B \times H) \times (\alpha'/L)] \qquad (16)$$

The right-hand side of proportion (16) corresponds to the modulus of elasticity E and is determined through bending tests. This leaves parameters G, l, b, h and the deflection U at midspan. If four parameters are specified, the fifth one can be determined. Accordingly, one of the five parameters in proportion (16) can be calculated, if four parameters are specified. If U is unknown, for example, taking into account eq. (14) and the expression for the moment of inertia $I = b \times h^3/12$, proportion (16) can be transformed into the familiar formula

$$U = (G \times l^3)/(48 \times E \times I). \qquad (17)$$

Based on this principle, the dimensioning problem in terms of the fitness for purpose for structural and machine constructions can be solved for all load types. Gerstner summarised the "elastic force ratios" – i.e. moduli of elasticity E – derived from his main tensile and bending tests in a table (**fig. 11**), with E specified in the unit *Lower Austrian pounds per square inch*. For converting these values to N/mm^2 they have to be multiplied with 0.0081. The modulus of elasticity for rolled iron is thus

$$E = 24\ 109\ 000 \times 0.0081 = 195\ 283\ N/mm^2 \qquad (18)$$

and the average modulus of elasticity for wrought iron E = 178 882 N/mm^2. For structural steel a value of 210 000 N/mm^2 is used today. The value from eq. (18) according to Gerstner is therefore 7 % smaller than the value for structural steel. The deviations of the moduli of elasticity for wrought iron, rolled iron, piano wires, ordinary iron wires and steel mainsprings in Gerstner's table are predominantly due to the fact that the chemical composition, the material structure and the production technologies for these materials were very different. In Germany, classification of iron and steel did not commence until after 1876 (Kurrer 2003, pp. 796-7).

Pag.	Nro. des Versuches.	Materie.	Querschnittsfläche F in □''	Elastisches Kraftverhältniss $\frac{Q.L}{F.\alpha'}$.
339	1	Eichenholz.	1	1463000
do.	2	do.	1	1225000
340	3	do.	1	1116000
do.	4	Buchenholz.	0,9424	1228000
341	5	do.	0,9024	1503000
do.	6	Fichtenholz.	1	1808000
342	7	do.	0,8728	2089000
do.	8	do.	0,9184	2250000
343	9	Tannenholz.	0,9750	1668000
do.	10	do.	0,9425	1299000
344	11	do.	1	2574000
349	1	Gusseisen.	1,7590	12100000
do.	2	do.	1,7590	11171000
350	3	do.	1,8338	11658000
do.	4	do.	1,7736	10488000
351	5	do.	1,8601	11257000
do.	6	do.	1,9468	12065000
352	7	do.	2,0565	11092000
353	1	Schmiedeeisen.	0,7007	22759000
354	2	do.	0,6332	21440000
do.	3	do.	0,4117	23630000
355	4	do.	0,6717	20508000
356	5	Gewalztes Eisen.	0,6484	24109000
266	1	Clavierdraht.	1/2037	22620000
267	2	do.	1/4382	23581000
268	3	do.	1/3099	25170000
do.	4	do.	1/1046	18618000
269	5	do.	1/598	19981000
do.	6	Gemeiner Draht.	1/412,5	16454000
270	7	do.	1/258,3	21999000
do.	8	Stählerne Uhrfeder.	1/747,5	21172000
271	9.	do.	1/480	16236000

Figure 11. Synopsis of Gerstner's tensile and bending tests (Gerstner 1833, p. 384)

The modulus of elasticity not only has an empirical side, but also a theoretical one. This paper focuses on the role of the modulus of elasticity in engineering practice, for which Navier and Gerstner provided the scientific basis, rather than on the implications on the theory of nature (Kahlow 1990) or on the history of its invention (Beal 2000). Like Gerstner, Navier recognised the central role of the modulus of elasticity – in parallel with tensile and compressive strength – as early as 1826 (Navier 1826). The main difference was in their technique:

- While Navier placed Young's definition of the modulus of elasticity right at the top of his structural theory (Navier 1833/1878, p. X), he regarded it as a material constant (Navier 1833/1878, p. XII) and not as proportion as set out in eq. (14) by Gerstner.
- In his structural theory Navier used the modulus of elasticity as a techno-scientific term, i.e. for generalising experiments in terms of a material constant. Gerstner, on the other hand, related proportion (14), which is equivalent to the modulus of elasticity, to a single experiment.
- For wrought iron, for example, Navier generally used an average value 200 000 N/mm^2 derived from numerous experiments (Navier 1833/1878, p. XII).
- Navier derived the formulas for the deformations due to beam bending – such as eq. (17) – directly from the linearised differential equation for beam bending and was therefore also able to solve statically indeterminate problems (Kurrer 2002, pp. 206-9).
- Navier consistently moved away from the theory of proportions – established by Galilei in 1638 – in terms of beam theory in particular, and theory of material strength in general. Gerstner, on the other hand, continued to refine his empirical theory of proportions in his theory of material strength and structural design.

At the level of elastic beam theory, Navier achieved a synthesis of the theory of material strength and structural analysis in the form of structural theory. In Gerstner's work, the relationship between the theory of material strength and structural design is extrinsic (see **fig. 4**):

> Structural design is based on rules determining the strength of buildings in terms of the thickness of the individual components and their arrangement. In the previous chapters (on the theory of material strength – author's note) we have already established the theory of material strength for the individual components of a building. All that is outstanding for structural design are therefore laws for their arrangement.
>
> (Gerstner 1833, p. 385)

In contrast to Gerstner, Navier did not discuss separate laws for the arrangement of individual structural components to form the overall structural system. In fact, an integrating analysis of the complete structural system only became possible with the advent of computer-aided mechanics (computational mechanics).

Gerstner thus laid the historical and methodological keystone for the preparation period of structural theory (1575-1825), while Navier laid the cornerstone for the discipline formation period of structural theory (1825-1900).

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