

How Stiff Is a Curved Timber Plank? Historical Discussions about Curved-Plank Structures

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The 120 foot (approximately 38 metres) wide timber cupola of the *Halle au blé* in Paris quickened interest of European engineers and architects at the end of the eighteenth century. The cupola, which was economically assembled from small, thin planks, was designed by the architects Legrand (1743-1808) and Molino (1743-1831) and erected by the master-carpentry Roubo (1739-1791). With their design the building masters resurrected at the beginning of the nineteenth century a structural idea dating from Renaissance times. The two-hundred-year-old construction methods, however, were almost completely forgotten. A tradition of how to build a curved-plank structure properly had not developed over the intervening time.

The curved-plank cupola of the Halle au blé stood up to the designs of solid stone cupolas. Its structural lightness asked emulating building masters a lot of questions; moreover forced them to think in new directions:

- On the one hand they had to realise that the laws of shape and mass of traditional European masonry vaulting structures are not directly applicable to curved wooden planks.
- On the other hand the use of the historical, light timber structure forced a competent handling of material strengths, elasticity, as well as a knowledge of load transmission within the relatively soft carpentry joints. These requirements were hardly able to be fulfilled using the laws of material sciences and mechanics of that time. To meet these requirements causes problems even today for engineers when they are calculating existing, often very filigree curved-plank structures.

But which methods did the building masters used to guarantee structural safety 200 years ago?

The first part of this paper deals with the history of curved-plank structures in Europe and the handed-down structural experiences, which provided the basis for the reintroduction of the construction method around 1800. The second part introduces the structural discussions about the curved planks at the beginning nineteenth century in Germany based on the analysis of the technical literature of that time.

Experiences with Curved-Plank Structures before 1800

The curved planks were made of two or more layers of thin timber planks, in a vertical plane, which were nailed together in a curved shape by treenails. The idea of these assembled curved timber

elements had already been used for centuries in the building of centering for masonry arches and vaults (Fig.2), for waterwheels and in shipbuilding. Completely planked, curved timber roofs covered the Renaissance palaces of Vicenza and Padua. Already Leonardo da Vinci (1452-1519) had introduced assembled, toothed curved planks in his notes, published in the Codex Atlanticus (Fig.3). He also researched the load bearing capacity of differently shaped arches (Fig.4). The Italian architectural theorist Sebastiano Serlio (1475-1554) took up the idea of the curved roof structures as well in his seventh book on architecture (Fig.5) published in 1619 after his death. However apart from these drawings, no structural building guidance can be found in either of these works.

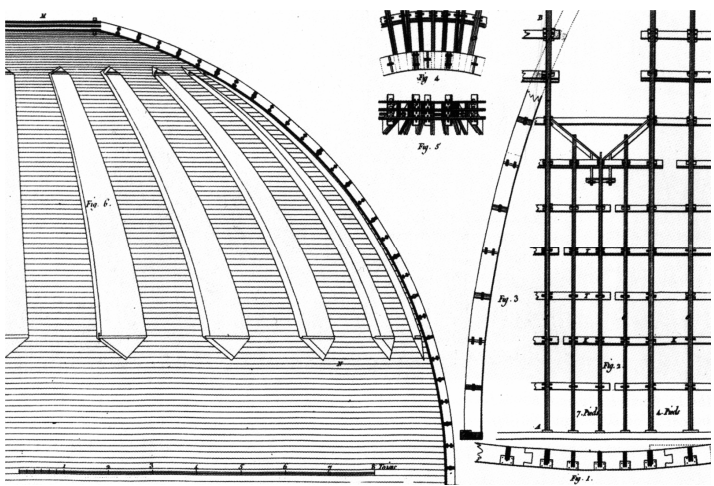


Figure 1. Timber roof structure of the Halle au blé (Krafft 1805, plate 71)

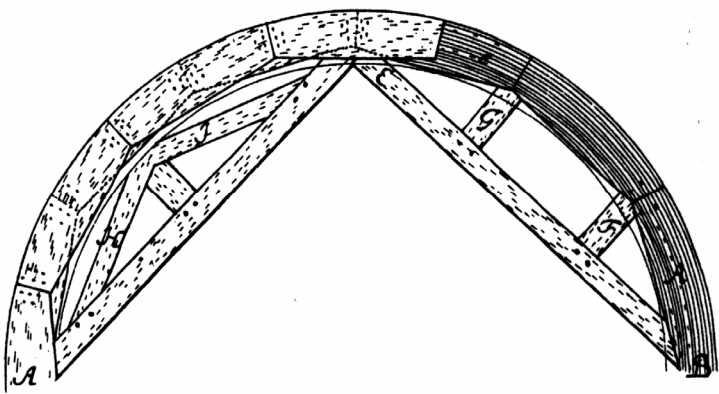


Figure 2. Curved planks in vaulting centring (Leupold 1726, tab. 18)

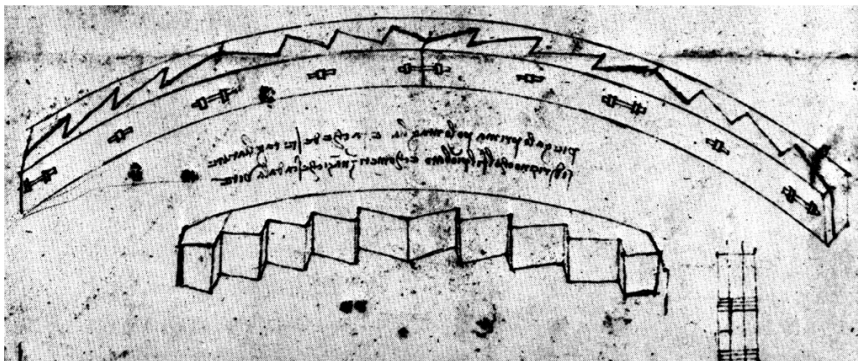


Figure 3. Toothed curved rafters designed by Leonardo da Vinci (Reti 1996, p.267)

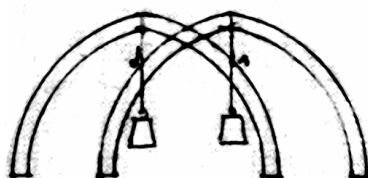
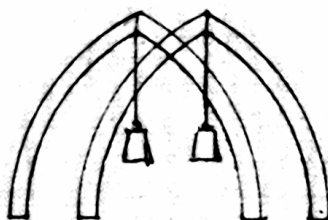
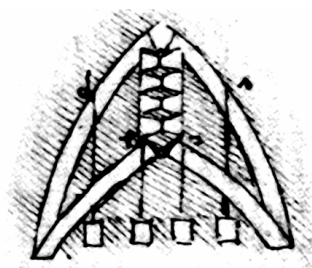


Figure 4. Structural studies of Leonardo (Reti 1996, pp.213-4)

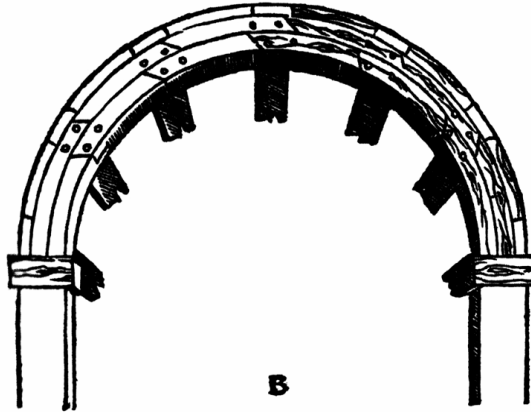


Figure 5. Timber trusses of Serlio (Serlio 2001, p.349)

The French Philibert de l'Orme (1514-1577), architectural theorist and architect of the Royal court of Henry II, was the first who worked more deeply on the construction of curved-plank structures in his *"Nouvelles inventions pour bien bastir et à petits fraiz"* (l'Orme 1561) (Fig.6). On what experience, though, did de l'Orme base his work about curved planks?

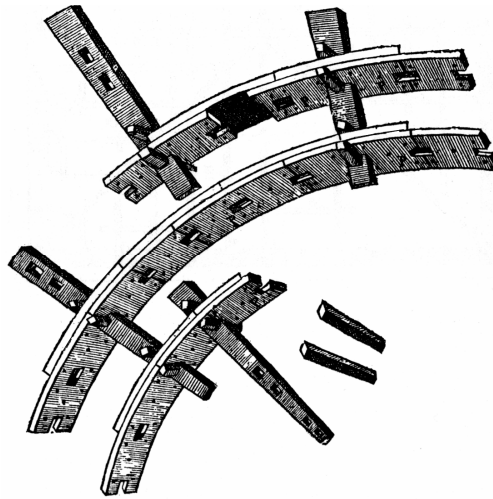


Figure 6. Curved-plank structure of de l'Orme (de l'Orme 1561, p.286)

Philibert de l'Orme was an expert in the classical works on architectural theories of his time. He undertook educational journeys in neighbouring countries, such as Italy. His architectural background was based on masonry structures. In this subject he devoted himself to the geometrical aspects of stone cutting in vaulted structures. He adapted his experiences from stone structures to

timber structures. Compared to masonry structures, de l'Orme considered the lightness and the small horizontal shear forces at their footings, the main advantages of his timber structures. He abandoned the complete timber planking which was used in the Italian Renaissance palaces and placed the curved planks, consequently edgewise. Furthermore he introduced the interlocking joining of many curved planks - *les liernes* [Engl.: the ribs]. He suggested using only short and rigidly fixed curved planks – the shorter the planks the stronger the whole structure, provided that the joints were made rigid enough. He did not regard the joints as weak parts at all.

De l'Orme favoured the semicircular profile for curved-plank structures. He felt absolutely confident about their load bearing capacity and proposed spans up to 400 metres (Ruesch 1997, p.11)! However, de l'Orme himself only built curved-plank structures spanning up to 19.5 metres (Meschke 1989, p.52). What made him believe in the great stability of these structures remains unclear. He convinced his clients of the general load-bearing behaviour of curved planks by public load tests. In the presence of the King he once tensed the curved planks of a flat ceiling using two screws fixed to the floor. He screwed them down until the flooring started to lift. Because he did not notice any deflection of the curved planks, he considered this proof of the huge bearing capacity of his invention (Gilly 1797, p.75). Likewise he reported loading tests on the castle La Muette (**Fig.7**) where, according to his statements, he got good results throughout (Gilly 1797, p.5).

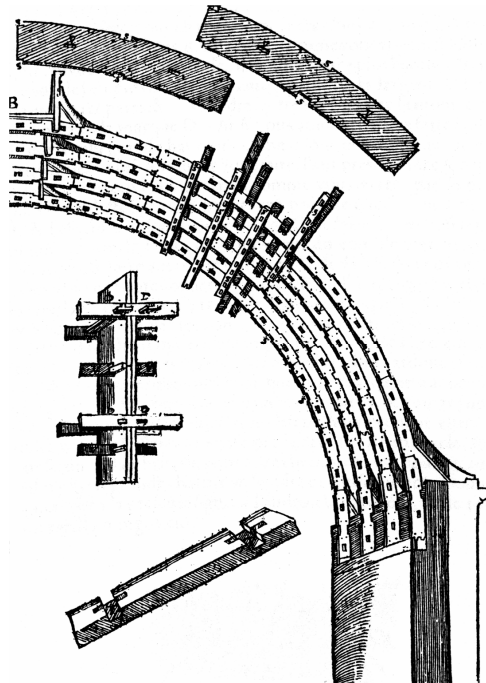


Figure 7. Roof structure of La Muette (de l'Orme 1561, p.293)

The tests by de l'Orme served the promotion of his curved-plank structures better than the scientific understanding of their stability. Important details of the experimental set-up, cross sections of the structural members, applied forces and the strengths of the counter bearing walls all remained untold. This may explain the high deformations which appeared only a short time after the erection of one of his test objects - the castle La Muette (Meschke 1989, p.82).

De l'Orme tried to spread his structural idea through publications and the demonstration of models of curved-plank structures throughout Europe. Despite these efforts they were not taken up in the literature on timberwork after de l'Orme's death. Only a few curved-plank structures were built in the following years. Building masters at the end of the eighteenth century only had de l'Ormes descriptions and drawings to inform them. The constructors of the Halle au blé had to rely on the two-hundred-year-old prints in de l'Orme books.

First Theoretical Considerations of the Stability of Curved Planks, in Early Nineteenth Century Germany

The first German publications on the construction of the curved planks, around 1800, described the construction of the cupola of the Halle au blé and referred to the original documents of the inventor de l'Orme.

The first comprehensive German work which dealt with the construction of curved planks was published by the architect David Gilly (1748-1808) in 1797. Beside a historical overview of the construction method the work presented a commented, German translation of de l'Orme's essays on curved planks. Gilly completed his translation by adding his own theoretical ideas. He too considered the curved planks were just a lighter adaptation of rigid stone vaults. Also a close colleague of Gilly, the engineer Johann Albert Eytelwein (1764-1848) regarded the timber joints as rigid connections. He neglected bending and sagging of the joints (Ardant 1847, p.VI). Elastic deformations were regarded as the results of existing pressure only (Ardant 1847, p.VI).

Gilly selected the shape of the curved planks according to the minimum resulting horizontal shear forces at their supports. He considered the Gothic pointed arch the shape of greatest stability and minimum horizontal shear. Deducing from the questions of arch structures he undertook more precise theoretical investigations of shear forces existing at the supports and the identification of the places of greatest force (Gilly 1797 p.60). Until the mid-nineteenth century engineers regarded the shape of masonry arches, mainly optimised for the transmission of compression forces, to be the best shape for curved timber planks too. In 1825 Johann Michael Voit (1771-1846), the Royal building surveyor of Bavaria, also referred to the precedent of masonry structures (Voit 1825). Panzer (1835), a Royal government building officer in Bavaria, and the architect Johann Andreas Romberg (1806-1868) (1850) recommended that curved-plank rafters should take the shape of an inverted catenary.

Gilly promoted the structure as both lightweight and economical, but rigid. Convinced of the bearing capacity of curved planks Gilly condemned the *liernes* of de l'Orme's earlier years (Gilly 1801, p.28). He even thought the curved rafters would be weakened through the interlocking of the *liernes*. He thought the curved planks should be stiffened as was usually done in roof structures at that time using the roof hip, gable walls, roof battening and cross laths. In his first publication in 1797 Gilly adopted the two-hundred-year old span-wide-differentiated dimensions of curved rafters given by Philibert de l'Orme. Except when discussing the influence of the height of curved rafters on their load-bearing capacity, he made no recommendations concerning the heights of rafters. Rafter heights were missing in de l'Orme's work as well.

Jacob Christian Gustav Karsten (1805), a lecturer at the Rostock Academy, published an essay on the construction of curved planks. He compared the load-bearing capacity of curved and linear roofs and also the influence of their rise on wind loads and the resulting compression forces. He declared that roofs of parabolic shape were the strongest (Karsten 1805, p.31). Like Gilly, Karsten referred to the results of Pieter von Musschenbroeck (1692-1761), Professor of physics at Leiden University, who measured the strengths of many construction materials in the 1730s. From the strengths Karsten deduced formulae for calculating the necessary height of curved planks in which the resistance varies as the square of the height of the curved planks. But Karsten considered only the single curved planks spanning between their ends; he regarded timber carpentry joints as fixed joints. From the inverse proportionality of span and bending stiffness, he inferred the advantage of using many and small curved elements. According to Karsten's calculations a curved rafter was eleven times stronger than a linear rafter (Karsten 1805, p.77).

First Practical Experiments with the Curved-Plank Structures in the Nineteenth Century

After popularising the curved-plank structure and its use in several buildings it was possible to study their general load-bearing behaviour and to correct any deficiencies found. Based on these studies, David Gilly revised and completed his instructions on how to build curved planks. Already in 1801, four years after his first book on the subject, he suggested a minimum height of curved planks of 10 inches (25.5 centimetres) (Gilly 1801, p.6). At the same time he increased the necessary width. In 1805 he spoke about meeting the architect Legrand in Paris (Gilly 1805, p.88) whose opinion about the load-bearing behaviour of curved planks was that: "they behave well, but should be monitored, because they sway". In 1811 Gilly reported on the failure of several curved-plank structures after heavy windstorms (Gilly 1811, pp.127-8). He traced the cause of the failures back to their unfavourable shapes, insufficient stiffening and poor carpentry skills. He described typical deformations of curved plank structures such as bulging (**Fig. 8**) and lateral displacements (**Fig. 9**). On the basis of these observations Gilly retracted his earlier statements about disregarding the *lierne* stiffening used by de l'Orme and advised attaching additional lateral stiffening of scarf-jointed beams and using tension rods to prevent bulging (Gilly 1811, p.131].

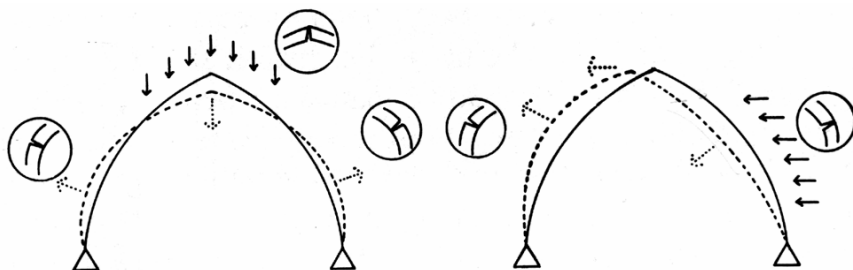


Figure 8. Typical bulging of curved-plank structures (Ruesch 1997, Fig. 13)

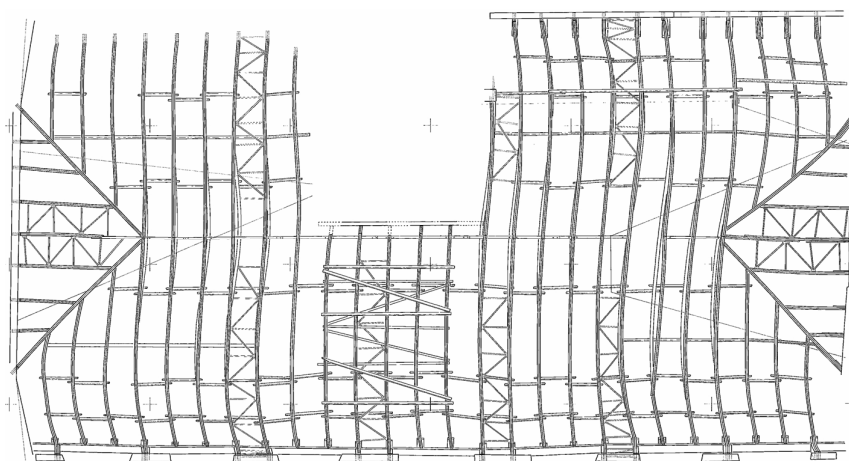


Figure 9. Typical lateral displacements of curved-plank structures (ARGE Hochofenhalle 2002)

The German technical literature also published critical statements of the French architect Rondelet (1743-1829). Already in 1805 he demurred about the bearing capacity of the curved planks (Gilly 1805, p.88). According to him they lose their stability in structures spanning more than ten metres. Rondelet recognised the stability arising from the elasticity of the curved planks. He explained that an assembled beam made of many small elements is not able to reach the bearing capacity of a solid beam. Wider spans would therefore need very large cross-sections that were difficult to obtain. Even the Halle au blé was only possible to erect by the use of many very strong planks. Indeed, in this respect, the Halle au blé could not be called a timber-saving structure at all.

Franz Xaver Johann Maschek, a student of the mathematics and statics professor Franz Joseph Ritter von Gerstner (1756-1832), worked on the analysis of the load-bearing capacity of curved-plank bridges, which were shaped according to the thrust line. He too criticised the stability of the plank-joints (Maschek 1843, p.92). He attributed the lack of stability in the joints to the deformation

of the timber under compression forces. Deformation of the arches caused them to deviated from the intended line of thrust which, according to Maschek, had caused the failure of some existing structures. He suggested pre-compressing the thrust-line-shaped planks before they were assembled to make bridge structures (Maschek 1843, p.135).

Curved-Plank Assessments on Load Tests at the Beginning Nineteenth Century

Gilly reported a load test of a curved-plank arch 60 feet (18,84 metres) long, 7 feet (2.2 metres) high and 2 inch (5.2 centimetres) wide, made of two layers of planks (Gilly 1800a, p.134). Under the load of roofing materials, no deformations were measurable. After these results King Friedrich Wilhelm III issued the command to repeat the load test on a bridge of same length and 20 feet (6.28 metres) wide, consisting of 5 curved-plank arches. Details of these tests were not published.

Franz Ernst Theodor Funk, a hydraulic engineer in the kingdom of Westphalen, erected the six spans of the 96-metre long Weser bridge (Bunte Bruecke) near Minden in 1800 (**Fig. 10**). After twelve years he investigated the structural performance of the bridge and found it had developed no problems at all (Funk 1812). After describing the structure of the bridge in detail he considered the loads that it had carried over its twelve-year life. In this connection he discussed the general state of structural calculations of curved planks. His dissatisfaction with the common approaches structural analysis he concluded that the bending strength of curved planks can only be found by load tests. He tested scale models of two curved planks to measure their absolute bearing capacity and published the description and analysis of his tests (Funk 1812). The two curved planks followed the shape of the bridge girders and were built to a scale of 1 inch to 1 foot. They were placed 24 inches (60 centimetres) apart and connected by boards. The curved-plank rafter footings were fixed in abutments. Loads were placed on the peaks of the two rafters and increased until fracture after 21 increments. The deformations under increasing loads were also measured. The curved rafters always failed at their joints. Funk therefore proposed strengthening the joints by additional planks. The analysis of the tests allowed Funk to criticise the statements of Karsten about the bearing capacity of curved- plank structures. Funk converted his results of the model tests to full-size structures using proposals by the professor in mathematics Johann Spaeth (1759-1842) who had worked on the statics of timber-arch bridges and analysed several materials tests (Spaeth 1811). He proposed converting the results obtained in the model tests to full scale structures on the basis of the resulting stresses. By assessing a real structure and the loads it carried, and undertaking additional model tests Funk was the first person, both to assess the safety of an existing structure, and to provide others with a detailed basis for assessing the strength and safety factor of similar curved-plank rafters. Funk advised using a safety factor of 3.4 for ultimate loads (Funk 1812, p.33).

Zimmermann, a building officer in Lippstadt, criticised Funk's analysis, particularly his poor documentation (Zimmermann 1830). Before he built a curved-plank structure himself, Zimmermann tested scale models at a scale of 0.5 inch to 1 foot. He tested five single standing

arches with centre angles between 40° and 60° as well as two arches which were connected with each other. The rafters had fixed supports. Seven ropes were fixed above the axis of the arches. The ropes carried a hanging board. Sandboxes and individual loads were equally distributed on the board. The total elasticity of a curved-plank structure was tested by a single load at the crown of one of the two connected arches. He measured the deflexion of the crown to an accuracy of $1/16$ inch with the help of a rope tangential to the arch. The timber elements themselves never broke; failure was always in the lower joints (**Fig.11**). This is why Zimmermann requested strengthening of the supports of the curved rafter using bigger cross sections. For loads more than $2/3$ of the ultimate loads, the curved rafters needed lateral support. This confirmed the low lateral stiffness of the curved planks, which was found to be one of the main problems of poorly stiffened curved timber roofs. Zimmermann recorded in detail the deformation behaviour until failure. He detected the more elastic structural behaviour of the curved planks compared to masonry arches, and compared it to the behaviour of iron screwed frames or tubes. He recommended using a safety factor of 5 for the ultimate loads. He also reported that his survey of the Bunte Brücke, built by Funk 30 years earlier, found the bridge still in excellent structural condition.

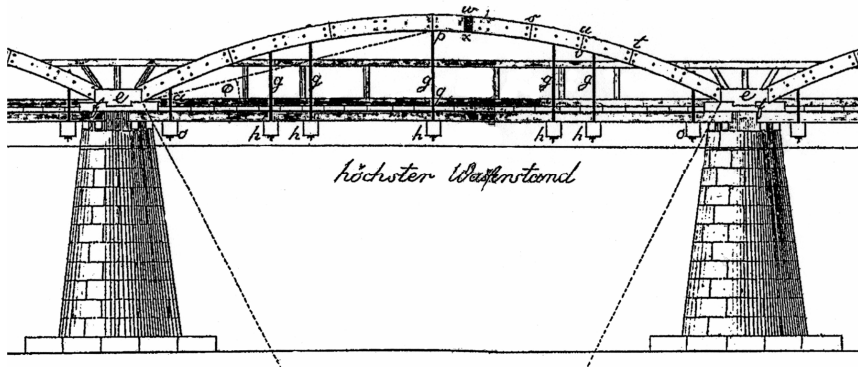


Figure 10. Bunte Brücke built by Funk (Funk 1812, Fig. 1)

In 1840, the French engineer and professor of the art of building and construction at the engineering and military school in Metz, Paul Joseph Ardan (1800-1858), analysed common strutted frames and timber arch structures. By building the timber arches in de l'Orme's manner he already recognised their great flexibility. After these experiments he strictly distinguished the behaviour of masonry arches from those of iron or timber. Furthermore he recognised the great influence on the bearing capacity of curved planks of both the number and the strength of the connections between elements. By analysing existing test results he emphasised that it is not enough to find the ultimate loads on the tested elements; rather it was equally important to consider the degrees of moment.

Ardan analysed the deformation of arches on small scale models made of solid timber voussoirs. The performance of the arches under different loads laid the basis for further load-bearing tests on

full-scale models. He also determined the modulus of elasticity using test prismatic pieces of pine timber 7 centimetres in length. The large scale models of curved and linear rafters had spans of 12.03 metres and the arches were restrained to prevent lateral bending. In addition to general results about the bearing capacity and elasticity of the curved planks, the tests established the lateral thrusts at the supports by adapting the principle that Leonardo da Vinci had already used for his researches on the horizontal thrust of arches and vaults (**Fig. 12**). The supports were mounted on cast iron rollers, rolling on rails. The arch bearings were held in position by adjusting the loads applied by means of ropes and pulleys (**Fig. 13**).

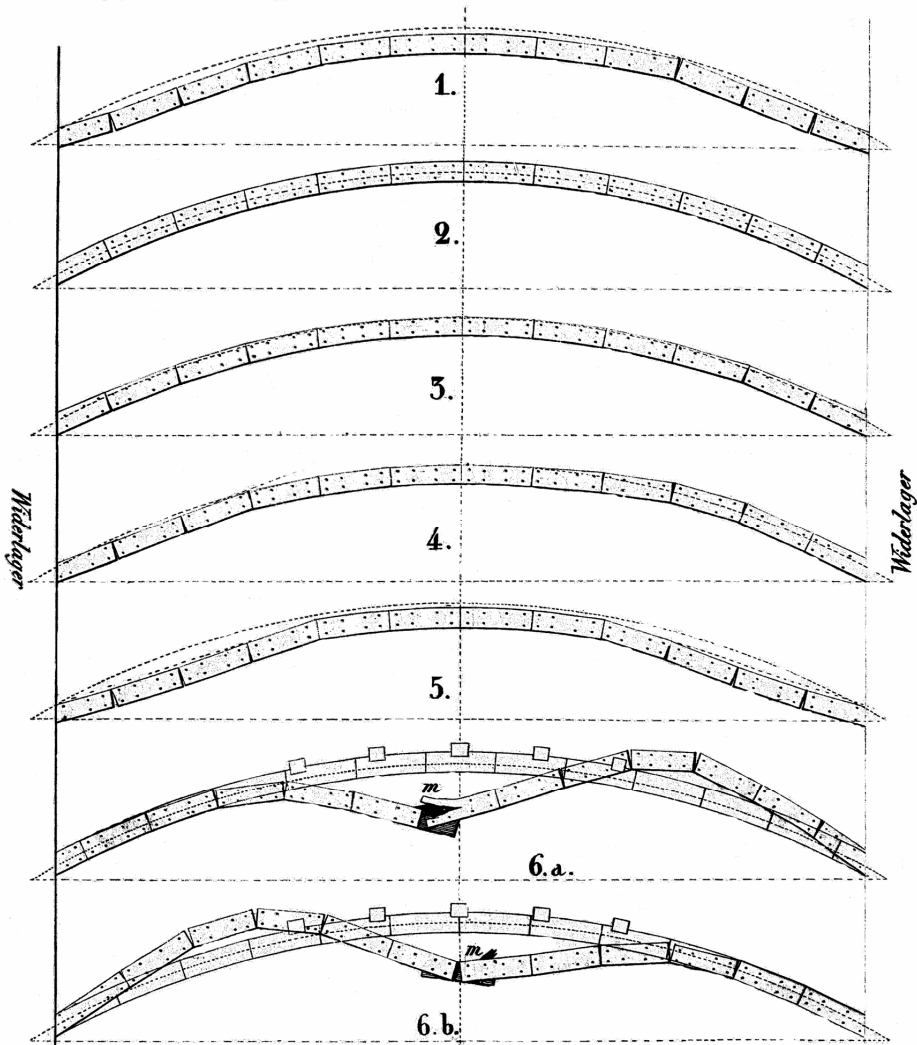


Figure 11. Model Tests of Zimmermann (Zimmermann 1830, Fig. 10)

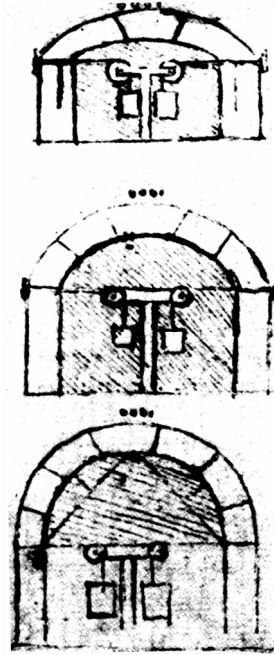


Figure 12. Tests by Leonardo da Vinci on arch stress (Reti 1996, p.212)

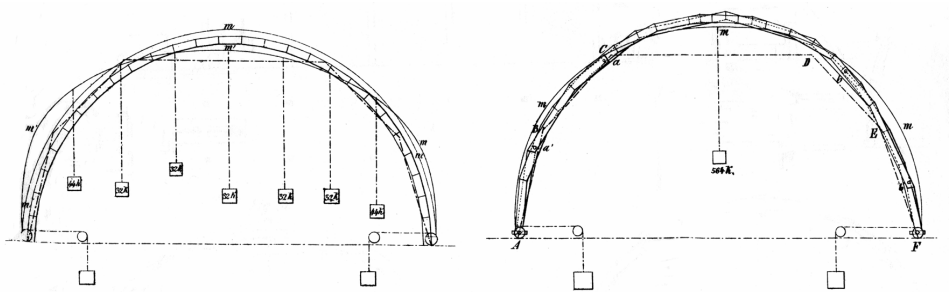


Figure 13. Model Tests of Ardant (Ardant 1847, Fig. 23,24)

Ardant was the first to use the theory of elasticity of Henri Navier (1785-1836) in the analysis of curved-plank arches. He gave a basis for calculating and dimensioning curved planks based on the statics of linear rafters. He recognised that the assembled curved planks had only half the bending resistance of solid curved timber elements of the same length and curve. The ultimate strength was even lower. For the additional safety factor he distinguished between short-term and long-term application of loads to structures. In the short term, timber structures might be safely loaded up to $\frac{1}{4}$ of their ultimate strength while for long-term loading, it was safe to load only to $\frac{1}{8}$ of their ultimate strength.

In 1847 the works of Ardant were published in German (Ardant 1847). The disastrous results concerning the load-bearing capacity of curved planks spread quickly among the architects and engineers and initiated the decline of this type of structure. In the following years only a few curved-plank structures were erected in Germany. Nevertheless, some 20 years after Ardant's publication models were still being used in load tests to solve questions about the structural behaviour and design of these structures, for example the 12-metre spanning curved-plank roof of the Bourse of the Berlin cattle-market (Fig. 14).

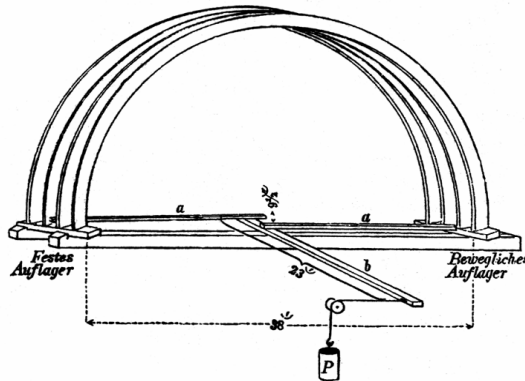


Figure 14. Loading test at the Bourse of the Berlin cattle-market (Orth 1872, Fig. 1)

CONCLUSION

German building masters adopted the curved-plank structure at the beginning of the nineteenth century. The curved structures were especially appealing because of their supposed economy in using timber. However understanding of mechanics and the knowledge of material strengths were insufficiently developed to enable engineers to determine load-bearing capacities using structural theory alone. This type of structure lacked a strong tradition of practical construction and the basis of curved-plank roofs in the early nineteenth century was still the two-hundred-year-old work of the French architect Philibert de l'Orme. Throughout the first theoretical publications in Germany curved planks were regarded as almost rigid arch structures. Therefore experience of masonry vaults was adopted directly for the timber structures. Aspects of elasticity were totally neglected. Questions appropriate to masonry arch building, such as the ideal shape of the arches and the resulting bearing reactions, dominated the early discussions on curved planks. Only after observations of the behaviour of the first real structures was it apparent that elasticity was vitally important, and that timber arches are fundamentally different from masonry arches. After the first failures, stiffness and stiffening became more important issues for the building masters. Besides load tests after the erection of new curved-plank structures, scale-model tests were used to gain information on the load-bearing capacity of the assembled structural elements. It took German building masters half a century to recognise the true behaviour of timber curved planks and to

achieve safe results based on the existing stresses. The realisation that the elasticity of curved planks was of vital importance was also the starting signal for their decline.

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