The Early Dome of Sant'Alessandro in Milan (1627): a First Study of the Behaviour of the Structural Core with a Dome Resting on Four Free-Standing Pillars

Irene Giustina, Elide Tomasoni and Ezio Giuriani

The great difficulties in the past that the construction of masonry domes caused architects and builders in particular seemed to sharpen in the religious buildings characterized by a plan in which the most important space was covered by a major vault or dome supported by four free-standing pillars, joined by four main arches and four pendentives and connected to the perimeter walls by proper transverse and longitudinal vaulted systems. As widely known examples, there can be cited the statical problems that arose in the Justinian basilica of Hagia Sophia in Constantinople from its first building phases (Mark, Cakmak 1993; Mainstone 1997), as well as those in the neoclassical church of Sainte-Geneviève in Paris, faced first by Jacques Germain Soufflot and then by Jean Baptiste Rondelet with difficult strengthening interventions (Bergdoll 1989, also for further references). Referring to the Lombardi architectural culture of the late-sixteenth century, the example of the wide debate concerning the reconstruction of the domed core of the Palaeochristian basilica of San Lorenzo in Milan is well known (Ferrari 1771; Rocchi Coopmans de Yoldi 1991; Giustina 2003).

A particular case of this structural articulation was presented by the *quincunx*, the central plan probably of Armenian derivation, spread in the Byzantine architectural culture between the midninth and the mid-twelfth century and which appeared in Italy from the early Middle Ages (for example the Carolingian sacellum of S. Satiro in Milan; Sannazzaro 1992). The *quincunx* gained great success in many regions of the Italian peninsula from the end of the fifteenth century, being then largely adopted by Bramante – raised to its apogee in the renewal of St. Peter in Rome – and used for centuries in religious architecture (Krautheimer 1986; Günther 1995; Bruschi, Frommel, Wolf Metternich, Thoenes 1996; *La chiesa a pianta centrale*, 2002, *passim*). That kind of plan was formed by a square divided by an inscribed Greek cross into nine smaller panels of which those at the four corners and the central one were usually covered by domes. The central space, the widest and most meaningful of the building, was covered by the major dome sustained by four free-standing pillars, four main arches and four pendentives.

The structural elements of the central core of the *quincunx* plan, their behaviour and their mutual interaction, especially in case of huge structures, greatly worried architects and builders, as it is shown by the difficult Renaissance renewal of St. Peter in Rome: Bramante himself, and then Antonio da Sangallo and Michelangelo were repeatedly concerned, probably conditioned more by statical reasons than by liturgical needs, with the shape, the sizing and the firmness of the pillars

and the main arches. Even more worrying was the damage found in the drum and the dome executed by Michelangelo and Della Porta, to which Giovan Battista Poleni gave repair in 1743-46 (Poleni 1748; Di Stefano 1980; Bruschi *et al.* 1996; Como 1997; also for further references).

The great interest of this subject stimulated the present work, which is set in the context of a wider study that intends to fulfil a first, general outline of the design aspects, the construction practice and the understanding of the structural behaviour of masonry domes in the architectural culture in Milan and Lombardy between the second half of the XVI and the first half of the XVII century (Giustina 2002a; Giustina 2002b; Giustina 2003; Giustina, Tomasoni, Giuriani 2004). Taking as a starting point the vicissitudes of the first dome of the seventeenth-century quincunx church of S. Alessandro in Milan, which was demolished after the appearance of deep cracks, the strengthening interventions and the subsequent plans designed, this paper aims to present a first study of the global structural behaviour of the central domed core resting on four free-standing piers. Following a scientific research methodology that bridges the investigation tools belonging to the historical disciplines, based on archival and documentary sources, with those more specifically belonging to structural engineering, such as finite element analysis. The first purpose of the present study is to understand the possible reasons that led to the demolition of the first dome of S. Alessandro and to assess whether the subsequent plans, if executed, could have avoided the problems previously shown. Attention is then paid to the system bearing the dome - the piers, the arches and the pendentives - and, starting from the analysis of the case of S. Alessandro, the behaviour of a masonry domed core sustained by four free-standing pillars is investigated.

Pointing out the structural mechanisms that set up in the system composed by dome, drum, arches, pendentives and piers enables useful general information to be provided about the efficacy of the structural interventions adopted in the past in those kind of domed cores, giving space to wider considerations about formal and planning choices – strictly connected to the structural ones especially in presence of that kind of system – spread since the mid-XVI century in the Lombardi architectural culture in relation to the profile and the exterior image of domes.

MAIN DESIGN AND CONSTRUCTION VICISSITUDES OF THE DOME OF S. ALESSANDRO IN MILAN

The church of S. Alessandro in Zebedia, of the Barnabite Order (Premoli 1913), was built in Milan from 1602. Its design was executed between 1600 and 1601 in Rome by Lorenzo Binago (1554-1629), Barnabite father "architectura artis bene peritus", who moved in Milan and pursued building construction until his death, in 1629. The wide range of Binago's activity – testified by at least twenty six Barnabite buildings in Italy and many other non-Barnabite buildings, by many unexecuted designs and architectural writings – was soundly based on Roman architectural culture and on technical knowledge strongly based on first-hand experience of building problems and on building site practice (*Lorenzo Binago* 2002; Repishti 2002; also for further references).

For S. Alessandro, Binago chose a *quincunx* plan (Fig.1) connected (adding a longitudinal expansion) to the fifteenth-century designs for S. Peter in Rome, of S. M. of Carignano in Genoa, of S. Lorenzo at the Escurial in Madrid (Zamboni 1778, p. 155), and to the plan of Pellegrino Tibaldi for the Cathedral of Vercelli. The same type of plan was proposed in many subsequent churches such as the Cathedrals of Brescia and Voghera, and adopted in other Barnabite churches such as S. Paolo in Casale Monferrato and S. Carlo ai Catinari in Rome (*Lorenzo Binago...*, 2002; Repishti 2002). The main body of the church was covered by five domes: four smaller ones at the corners and one larger, central dome resting – following the solution suggested by Bramante for S. Peter in the drawing Uffizi 20A – on four free-standing piers with an approximately triangular section. The piers were decorated along the diagonal sides with four pairs of giant granite columns, underlying the importance of the central space in the liturgy organization and in the architectural body of the church.

It is not easy to reconstruct the precise configuration of the central dome planned by Binago that was demolished in 1627 as a consequence of the deep cracks that appeared, because in the graphical documents it is not depicted in a consistent way. Probably, the drawings that more faithfully represent Binago's plan are two autographed transverse sections of the church, preserved in Milan respectively in the 'Archivio Storico dei Barnabiti' (ASBMi) and in the 'Raccolta Bianconi, Archivio Storico Civico' (RB, ASCMi) (Fig.2). The two drawings represent the dome as hemispherical and ribbed, crowned by a lantern, and covered by a "tiburio", the traditional solution of the Lombardi architectural culture. The dome, with a diameter of approximately 16 metres, is superimposed on a cylindrical drum with eight windows and is supported on the four triangular piers by means of the four pendentives and the four semicircular arches.

This dome had a very short life: the construction of the piers began in 1614, and by 1623 the eight granite columns and the arches were in place. The documents confirm that in 1624-1625 all the materials required to build the dome were ready and that the Barnabites intended to begin construction in 1626, as was, in fact, done. But in November of the same year the Fathers became very concerned about the firmness of the new dome, with the appearance of deep cracks in the arches. The Fathers thought that propping up the arches and reconstructing them using dressed stone to make them stronger could be sufficient to preserve the dome just finished. Advice was sought from many local builders but nobody could help. In February 1627 the damage to the dome was "data per certa e sicura da tutti" and it was decided to demolish not only the dome but also the arches and whatever else seemed necessary: "distruggere la sommità o il fastigio della detta chiesa fino alla cornice includendo anche l'arco e anche altro, se così ai periti, che si dovranno consultare nei giorni, parrà opportuno" (Giustina 2002a).

After the demolition Fabio Mangone (1587-1629) was consulted – a very active architect in Milan and, since 1617, architect-in-chief of the Cathedral of Milan. He gave his advice in 1628 in relation to the strengthening works on the load-bearing structure, in consideration of a new dome

construction. Mangone suggested increasing the section of the piers, joining them by means of masonry to the free-standing pairs of columns. He also advised strengthening the arches by doubling them "che si facessero doppi", nd increasing the section of the drum so as to make it rest entirely on the arches below (ASBMi, B, II, fasc. I; Giustina 2002a).



Figure 1. Lorenzo Binago, Plan of S. Alessandro, before 1606 (RB, ASCMi, VII, f. 6)

In 1629, after Mangone's death, Francesco Maria Ricchino (1584-1658) took over the building site of S. Alessandro. Ricchino was a highly esteemed professional in Milan and Lombardy, and rivalled Binago in many architectural works (Scotti Tosini 2003; Giustina 2002a; also for further references). Ricchino carried out Mangone's advice, provided the arches with chains (or ties - not precisely specified in the documents) and executed pointed arches above the existing round ones. Ricchino also carried out a new design for the dome, perhaps with Giovanni Ambrogio Mazenta (Giustina 2002a), a Barnabite father and practising architect (*Lorenzo Binago...*, 2002, pp. 45-90;

also for further references). The design is represented by two drawings preserved in Milan (RB, ASCMi): the new solution is very different from that "alla lombarda" carried out by Binago, with a hemispherical extrados, ribbed dome, superimposed on an attic and on a drum with windows (**Fig.3**). The extrados shape, which was extremely rare in Ricchino's architectures, was uncommon even in the local architectural culture between the second half of the XVI and the first half of the XVII century. It was very up-to-date, showing strong connections with Roman architectural trends, but, as the debate concerning the reconstruction of the dome of S. Lorenzo in Milan clearly showed, Lombardi architects and builders, who were familiar with the traditional "tiburio", were extremely adverse to entirely extrados domes, which followed Renaissance Roman and central-Italian models. They preferred, if necessary, partially extrados domes (Rocchi Coopmans de Yoldi 1991; Scotti Tosini 1999; Giustina 2003). This local architectural trend was clear since the first extrados domes had been proposed by Pellegrino Tibaldi, based on Roman precedent from the mid-XVI century, such as the dome of S. Fedele, planned as extradoxed and then executed with "tiburio", or the dome of S. Sebastiano, executed as extradoxed and then covered with a "tiburio" (Della Torre, Schofield 1994; Scotti, Antonini 2002).

In 1630, however, an outbreak of disease brought local building activities to a halt stop for a long time, and Ricchino's dome for S. Alessandro was never built. The dome was executed with an extrados shape only in 1693, designed by Giuseppe Quadrio, in a completely changed climate of local architectural and technical knowledge.



Figure 2. Lorenzo Binago, Cross sections of S. Alessandro (left: RB, ASCMi, VII, f. 12; right: ASBMi, Cartella Grande I, mazzo I, fasc. III)



Figure 3. Francesco Maria Ricchino, Cross section of S. Alessandro, 1629-30 (?), unexecuted (RB, ASCMi, VII, f.13)

FINITE ELEMENT MODELS OF THE DOMED CORE OF S. ALESSANDRO

The domes designed by Binago and Ricchino

To determine the causes of the damages that induced to demolish Binago's dome and to understand the structural behaviour of the demolished dome, of Ricchino's dome and of the structures below, finite-element analyses were performed on the basis of indications given by archival sources and of actual knowledge about the building materials and the technology used in Milan for the construction of domes. This allowed to better clarify the stress condition into such structural cores. Starting from design sections made by Binago and Ricchino, graphically revised with a cadinterface, two finite-element models have been constructed supposing the material to be linear elastic, isotropic and also able to carry tensile stresses. Brick 8-noded elements were adopted. The *Algor12* FE program was used and the following material mechanical properties were assumed: density, $\gamma = 1850 \text{ kg/m}^3$; Poisson's ratio, $\nu = 0.15$; Elastic Modulus, E = 5000 Mpa. (Fig.4).



Figure 4. Axonometric view showing the finite element models: a) dome designed by Binago; b) dome designed by Ricchino

This model helps develop an understanding of the three-dimensional behaviour of the structure, especially in the areas where the main arches, pendentives and free standing pillars meet, where diffusive effects caused by the thrust of main arches and the drum's vertical load could appear. It should be emphasized, however, that results obtained from such models of masonry structures, do not lead to a clear idea about the actual possibility of structural collapse: as is well known, after the formation of cracks in masonry structures, it is possible for alternatives equilibrium states to be established (Heyman 1996). The numerical analyses makes it possible to locate, in the studied structures, the first cracked area. As seen, this damage does not have serious consequences for the global stability of the structure, but it is quite large and, in the absence of a good scientific understanding, it might give serious concerns about the building's stability.

As a first step, only the dome and the drum are considered. The two cases examined are the hemispherical dome, corresponding to Binago's design (Fig.4a), and the pointed dome, corresponding to Ricchino's design (Fig.4b). The stresses in the domes obtained by means of the numerical analyses are similar to those determined with the elastic solution. The latter is significant for the structure before the cracking, according to classical membrane theory (Flugge 1973; Heyman 1977; Belluzzi 2001, III, pp. 245-249).

In particular, compressive stresses are shown everywhere along the parallels for the pointed dome (except for a small region near the springing, where the boundary conditions can cause some bending moments). Conversely, in the hemispherical dome, tensile stresses develop from the spring up to the keystone, almost. Note that these are predominantly membrane stresses, as they are the same in both the intrados and the extrados (Giustina, Tomasoni, Giuriani 2004, pp. 18-19).

The numerical analyses show that Binago's dome was probably affected by significant cracks passing through the masonry, resulting in considerable damage. On the other hand, Ricchino's dome, had it been built according to its design, would not have cracked.

We can conclude, therefore, that Ricchino's structural intuition to increase the rise of the dome in order to avoid the damage seen in the hemispherical dome was correct. This expedient, well-known in the past as a security device to reduce the thrusts on the buttresses, was proposed even though it was in contrast with the classical language that was fashionable in the seventeenth century: the classical language, necessary from a formal point of view, was recovered by Ricchino in the semicircular superstructure, with no structural function, located above the pointed dome.

Interaction between the two domes and the supporting system and evaluation of the structure's factor of safety

After investigating the behaviour of the domes, the interaction with the supporting system is considered. A first important result provided by the numerical analyses is the independence of the supporting system comprising the main arches, pendentives and free-standing pillars, from the geometry of the dome. The comparison of the analyses of the models of the dome and the drum and the analyses of the supporting system, also considering the structure below, have shown that the main arches, the pendentives and the free-standing pillars cause radial deformations in the drum. These deformations spread on the top of the drum, but they do not extend into the dome.

In the same way, the thrust of the dome spreads only in a limited area of the drum, because the drum hoop resists this thrust. The global structural behaviour is thus unrelated to the behaviour of the dome. Evidence for this is provided by the $_Z$ stresses along the external corner of the pillar, which are the most critical and develop on the most-stressed elements. Actually, the comparison between the structure with Binago's dome and the structure in which the dome has been replaced

with an equivalent vertical load, shows the same result with respect to stresses $_Z$ (Fig.5). This is because it is the drum that determines the hoop effect that resists the dome thrust.

The dome geometry does not modify the structural behaviour of the building and does not influence the tensional state of the pillars and the main arches.



b) model with the ideal dome without horizontal thrust and with an vertical load equivalent to Binago's dome

This result allowed further work to concentrate on the dome support system. To evaluate the interaction among main arches, pendentives and free-standing pillars and to estimate the structure factor of safety, a simplified model was considered. Such a model enables the structural problems of S. Alessandro's church to be explained and gives a basis for understanding the complex structural mechanisms in similar kinds of structure.

A static scheme has been defined for the structure designed by Binago. For the overall equilibrium of the section A-A (**Fig.6**) the system constituted by the slice AB (as wide as one quarter of the circumference), the corresponding drum BC, the pendentive CD and the pillar DE, was considered, in the first stage, as uncracked. This system was modelled as having a fixed-end in section E and a rigid support in C (ideally assumed to have unlimited stiffness). The force H_C is directed outward and arises from the drum resistance. The system is loaded by the self weight of the pillar, of the

pendentives, and of the quarter of the drum and dome. It is also loaded by the horizontal force $H_{\rm D}$ (Fig.6b). As will be explained later, $\overline{H}_{\rm D}$ is the lowest resultant thrust in the diagonal plane that the main arches exert on the pillars.

This structure is statically indeterminate in the uncracked stage. After the cracking, in the limit conditions, three plastic hinges form. The ultimate moment that can be resisted in the sections where plastic hinges develop is given by the vertical load multiplied by the maximum eccentricity. In the first stage, the restraint conditions of the structure determine bending moments in C and in D greater than the ultimate moment of resistance, and hence these sections will crack. In the second stage a plastic hinge arises in section E and the collapse mechanism develops (**Fig.6c**).

Applying the moment-equilibrium to the block D-E around the plastic hinge E, the ratio of the stabilizing moment (given by the pillar self weight and by the dome-drum system self weight multiplied by the maximum eccentricity) to the destabilizing moment (given by H_C and to \overline{H}_D) allows the safety factor, ψ , to be determined



Figure 6. Static scheme of the structural domed core with the free standing pillars

With reference to the line of thrust shown in Figure 6c, the moment-equilibrium of the pendentives CD around D gives the force H_C at the spring of the drum:

$$H_{\rm C} = \frac{P_{\rm C} \cdot e_{\rm C1} - P_{\rm P1} \cdot e_{\rm P1}}{h_1} = 173 \rm kN$$

(1)

where h_1 is 6,9 m and the vertical loads P_C and P_{p1} and the eccentricities $e_{C1} e e_{p1}$ are defined below.

For the evaluation of P_C one can assume that the weight of the dome-drum system, equal to 16,880 kN, is resisted in equal parts by the four pillars and by the four main arches. Therefore:

$$P_{\rm C} = P_{\rm D} = (1/8) \cdot P_{\rm TOT(dome+drum)} = 2,110 \text{ kN}$$

(2)

The pillar's self weight P_{p1} can be expressed as:

$$P_{p1} = h_1 \cdot A \cdot \gamma = 6.9 [m] \cdot 13.24 [m^2] \cdot 18.50 [kN/m^3] = 1,690 \text{ kN}$$

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where A is the pillar's area (A=13.24 m²) and γ is the density of masonry ($\gamma = 18.5 \text{ kN/m}^3$). The eccentricity e_{C1} of P_C and the eccentricity e_{p1} of P_{p1} , calculated with respect to the D hinge midpoint (Fig.6c), are:

$e_{C1} = 1.39 \text{ m}$ $e_{p1} = 1.03 \text{ m}$

These eccentricities are obtained by supposing uniform compressive stresses in the resistant portion of the sections E and C and by assuming the masonry's ultimate strength to be 2 MPa. (The assumption that the masonry has unlimited compressive strength is revised because the weight of the structure is considerable.)

The force Hc, given by the drum's resistance, is distributed to the pillar and causes, with the force \overline{H}_{D} due to the thrust of the main arches, the destabilizing moment M_{dest} .

The horizontal thrust \overline{H}_{D} is the lowest thrust resultant in the diagonal plane that the main arches exert on the pillars. The minimum value of the thrust occurs when a crack in the keystone and a sort of "natural resistant mechanism" over the main arches develop (Fig.7).



Figure 7. Possible natural resistant mechanism in the main arches



Figure 8. Axonometric view showing the virtual struts PD over the main arches

The natural resistant mechanism involves the development of the virtual struts PD (Figs.7-8). The maximum height, f, of the struts is 11m due to the presence of windows in the drum (even if the windows were absent, the struts PD could not develop up as far as the springing of the dome because they would have to pass outside the masonry section (Fig.8). Hence the minimum thrust, H_{D} , is:

$$H_{\rm D} = \left(\frac{P_{\rm D}}{2} \cdot l\right) \cdot \frac{1}{f} = 575 \text{ kN}$$

(4)

where l, approximately 6m, is the length of the strut PD in the horizontal plane and P_D is the load on the main arch, equal to one eighth of the total weight of the dome and drum. A vertical point load P_D , located at side of the window, is assumed (**Fig.7**).

The resultant thrust in the diagonal section is:

$$\overline{H}_{D} = \frac{H_{D}}{\sqrt{2}} \cdot 2 = 814 \text{ kN}$$
(5)

Hence the destabilizing moment on the pillar is:

$$\mathbf{M}_{\text{dest}} = \left(\mathbf{H}_{\text{C}} + \overline{\mathbf{H}}_{\text{D}}\right) \cdot \mathbf{h}_2 = 13,336 \text{ kNm}$$
(6)

Where $h_2 = 13.4m$.

The stabilizing moment is:

$$M_{stab} = (P_{p1} + P_{C} + P_{D}) \cdot e_{DE} + P_{p2} \cdot e_{p2} = 16,658 \text{ kNm}$$
(7)

Where:

$$P_{p2} = h_2 \cdot A \cdot \gamma = 13.4 \text{[m]} \cdot 13.24 \text{[m^2]} \cdot 18.50 \text{[KN/m^3]} = 3,282 \text{ kN}$$
(8)

 $e_{DE} = 2.18 \text{ m}$ $e_{p2} = 1.15 \text{ m}$ hence the factor of the safety for the structure is thus given by:

$$\psi = \frac{M_{stab}}{M_{dest}} = 1.25$$
⁽⁹⁾

From these results, it can be concluded that the structure designed by Binago, despite the deep cracks, was in stable equilibrium and therefore with a low likelihood of collapse.

Structural behaviour of the drum and of the "tiburio"

The horizontal force H_C , originating from the drum resistance, assures the equilibrium of the pendentive-pillar system, but determines drum deformation in the springing zone (Fig.9a).

The evaluation of the bending moments and of the stresses in the drum is not possible with a simplified model because an effect combination develops.



Figure 9. Deformation in the drum: a) horizontal section at the drum's springs ; b) axonometric view

The drum is loaded at the top by the shell's thrust and, at the bottom, by the force H_C . As the force Hc is greater than the horizontal dome thrust, it causes radial deformation (**Fig.9b**). In addition to this radial deformation, the drum is affected by an "arching" effect (typical of deep-beams). This effect is due to the different stiffnesses of the supports (**Fig.10**); in particular, the pendentives are stiffer than the main arches.

This conclusion is confirmed by the finite-element analysis which shows that, at the bottom, the vertical displacement of the drum next to the pendentives is smaller than it is adjacent to the main arches, while, at the top, the vertical displacement is approximately constant.

The different stiffness of the supports causes the development of the "arching". The stresses in these "natural arches" are shown in **Figure 10b.** (These stresses are equivalent to only about 10% of the total stresses, as shown by the finite-element analyses).

Hence, tensile stresses develop in the drum intrados adjacent to the pillars and in the drum extrados next to the main arches. These tensile stresses are significantly lower than the tensile strength of the masonry (Hendry, 1986).



Figure 10. Drum's model: a) vertical displacements; b) natural arches resistant mechanism

The finite-element analyses show that the keystone of the main arches and the area of the drum over the keystone are subject to membrane tensile stresses (Fig.11). In this region, in fact, the stresses in the keystone, due to the development of the struts PD, have to be added to the bending stresses due to the thrust of the pendentives. This is confirmed even by the historical documents of S. Alessandro.

The investigation also leads to an important conclusion regarding the function of the "tiburio", widely used in Lombardi architecture. This study, in fact, proves wrong the belief that the "tiburio", present in the drawings of the Binago's dome, can increase the structure stability. This element is effectively a surcharge to the haunch and, as a matter of fact, can produce a local thrust reduction near the spring. However, in the free-standing pillars system the "tiburio" increases the eccentric vertical load and, as a consequence, increases the tensile stresses in the arches and piers. Therefore, on the basis of this investigation, it can be concluded that the "tiburio" in domed structures with free-standing pillars, was built only for functional reasons and reflects the Lombardi builders' lack of familiarity with extradoxed domes.



Figure 11. Graphic of the hoop stresses along the drum's bottom (Binago's dome)

THE EFFECTIVENESS OF THE STRENGTHENING DEVICES USED IN S. ALESSANDRO

After the demolition of Binago's dome, many strengthening works were carried out in preparation for the construction of a new dome. The effectiveness of those works has been analysed with the aim of establishing whether, even based on simple intuition, the architects involved understood and correctly appreciated the real role of the devices proposed for strengthening the structure.

The cross section of the piers was increased by joining the free columns to the masonry standing behind (Fig.12), and the four arches were reinforced by inserting above each of them two pointed relieving arches, still visible in the masonry beneath the roof of the church.

Increasing the piers' section was clearly a good intervention; it increased the buttressing effect offered by the piers themselves, thereby increasing their stability. On the other hand, the effectiveness of the relieving arches is in doubt, though it followed well-established building practice that can be traced, for example, in the "tiburio" of Milan Cathedral (with stone arches built by Giovanni Antonio Amadeo; Ferrari da Passano 1986, pp. 208-209, Ferrari da Passano, 1988) and in S. Peters in Rome (with round arches built by Michelangelo, superimposed on the main arches; Poleni, 1748; Di Stefano, 1980): as demonstrated in the previous paragraph, the relieving arches

planned in S. Alessandro could not have a sufficiently high rise to reduce the outward thrust on the pier and assure its stability.



Figure 12. Fabio Mangone, Plan of a central pillar of S. Alessandro with strengthening interventions executed in 1628 by Ricchino (ASBMi, B, II, fasc. I)

It can be confidently asserted that the reduction of the depth of the presbytery and the width of the choir, planned by Binago (drawing ASBMi, Cartella Grande I, mazzo I, fasc. III, 1, 12) but unexecuted, was proposed as a strengthening device after the cracks had appeared in the domed core of the church, as was claimed in a recent study of the planning and building phases of S. Alessandro. Since Roman antiquity, it had been widely known, based on experience, that, in order to exert a better resistance to the thrusts of a vaulted structure, it was necessary to increase, and certainly not decrease, the sections of the resistant structure. And, as a matter of fact, the main part of the strengthening works carried out, or only planned in S. Alessandro, were directed to increasing the sections of resistant elements (Giustina 2002a).

As a consequence of the discussion in the previous paragraph, it can be concluded that to eliminate the cracks in the structure it could have been sufficient to insert horizontal extrados ties, able to resist the tensile stresses acting on the key of the arches, and vertical ties, to resist the tensile stresses acting on the piers (for analytic demonstration, Giustina, Tomasoni, Giuriani 2004, Appendix C).

While the use of vertical ties in S. Alessandro is not testified by archival sources, nor are traces of such ties visible in the church today, a number of horizontal intrados ties were introduced,

following correct structural intuition, although archival documents do not reveal who inserted these ties, or when they were fitted. One drawing does exist, probably executed by Mazenta and Ricchino, that shows the intention of inserting extrados ties "a braga" over the arches (Fig.13), a solution that, as mentioned above, would have been very effective. Underneath the roof of the church many bolt heads of dead ties over the arches can still be seen today, but it is impossible to discern whether they are those planned by Mazenta and Ricchino in around 1629, or whether they were placed at the end of the seventeenth century during the construction of the new dome.



Figure 13. Giovanni Ambrogio Mazenta and F. M. Ricchino, Plan of a new dome of S. Alessandro with strengthening devices, 1629 (?), probably executed (ASBMi, Cartella Grande I, mazzo I, fasc. III)

CONCLUSIONS

The present paper has studied the structural behaviour that generates tensile stresses at the key of the arches and on the piers, leading to the appearance of large cracks, in buildings that incorporate the structural system of a dome resting on four free-standing piers, similar to that adopted in S. Alessandro in Milan. The location of the most significant cracks, confirmed by the analysis carried out in the present study, agree with what was indicated in contemporary archival sources relating to

the uncertain stability of the dome of S. Alessandro. It has also been shown that those cracks, although very deep, would not have compromised the global stability of the structure, meaning that the painful demolition of Binago's dome was probably not necessary. The subsequent dome with pointed curvature, designed by Ricchino, would have led to a more restricted cracked area and would have generated a lower thrust on the drum, showing that Ricchino's structural intuition was sound.

The study also demonstrates, however, that the dome behaviour would not exert influence on the underlying load-bearing system, and that the change of curvature planned by Ricchino, based on widespread empirical knowledge, would have been substantially useless.

As the numerical models have shown, Binago's hemispherical dome, although developing deep cracks, was not the main cause of the damage to the structural core of S. Alessandro. The deep cracks at the key of the arches and on the piers were generated rather by the considerable load exerted on the arches and on the pendentives which were insufficiently buttressed.

Using simple statical schemes it has been possible to give reliable indications about the structural behaviour of the domed system resting on four free-standing piers, and to evaluate the structural factor of safety, thus providing a useful preliminary understanding of the behaviour of structures similar to the central domed core of S. Alessandro.

Finally, the simplified statical schemes helped define more precisely the structural role of the "tiburio" which is rather ineffective in domes supported eccentrically, by means of pendentives and arches, on four free-standing piers. It has been demonstrated that the "tiburio", widely used in Lombardi architecture, increases the tensile stresses at the arches and on the piers.

This research has extended application of the analysis and the numerical modelling techniques from the particular case of S. Alessandro to other structural systems of vaults and smaller domes, including *quincunx* buildings and other buildings with different plans but still incorporating a domed core resting on four free-standing piers, in which the outer vaulted elements can provide useful statical support to a central, domed core.

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