Analysis of the Statics of the Mycenaean *Tholoi*

Maria Teresa Como

INTRODUCTION

The architectural and structural definition of the Mycenaean *tholos* is still an open question, due to the lack of a complete survey caused by the structure's hypogean condition. In the archaeological literature the study of the tholoi mainly concerned the chronological classification based on the architecture's reading (Wace, 1921-1923; Pelon 1976). More recently, thanks to the visibility of some essential geometrical features of previously hidden parts, some authors (Gasche and Servais 1971; Wilkie 1992; Belli 1995) pursued studies that clarified the architectural process of construction. Among scientific and technical studies, some authors proposed a structural model of behaviour, investigating in particular the model of the so called pseudo-dome. Those studies lead to identify the dome's suitable profile able to ensure equilibrium by assuming a corbelled principle (Benvenuto and Corradi 1987). Donaldson was the first to understand the importance of the horizontal ring arch as an essential principle in the construction process (Stuart and Revett 1830). Cavanagh and Laxton (1981), using a multidisciplinary approach, connected the archaeological data to the static framework of the corbelling principle. Santillo and Santillo Frizell (1984) questioned their thesis, sustaining a different model of behaviour along the one of the masonry domes. Cremasco and Laffineur (1999) developed the static analysis of the failed dome of Thorikos by applying the finite elements' method in the context of the linear theory of elasticity. Cremasco and Laffineur pointed out the presence of tensile stresses along the ring at the dome's springer. These actions are not compatible with the corbelling principle.

The aim of the paper is to pursue a study of the *tholos* static behaviour, in relation to the architectural analysis. The investigation has been developed in three parts: the study of the architectural morphology of the monument, the analysis of a specific case-study –the Treasury of Atreus- leading to the construction of a complete geometric model, and the static analysis of the masonry dome.

THE ARCHITECTURAL FEATURES OF THE *THOLOS*

The Mycenaean domed tomb, generally known as *tholos*, is one of the most outstanding architectural structure of Late Bronze Age. The *tholos* reached monumental dimension as in the case of the Treasury of Atreus -with a diameter of 14.5 m-, which constituted the greatest masonry dome ever built by men before Roman domes.

The *tholos* is a circular chamber tomb covered by a masonry dome realised through the successive placement of cantilevered stone elements covered by a mound of earth and rubble stones at the extrados. The chamber room is obtained through excavation of a circular pit along a hill slope. The pit's depth matches the height of the entrance door's architrave (Holland 1921-23, p. 396); in this way the perimetric walls constitute earth retaining structures till the architrave level, while the remaining masonry structure lies above the pit creating, together with the covering mound, a characteristic volume, which emerges from the hill slope **(fig.1)**. The distinctive typology of the *tholos* lies therefore in the hypogean condition of the masonry dome inserted in the pit and surmounted by the emergent mound over the slope.

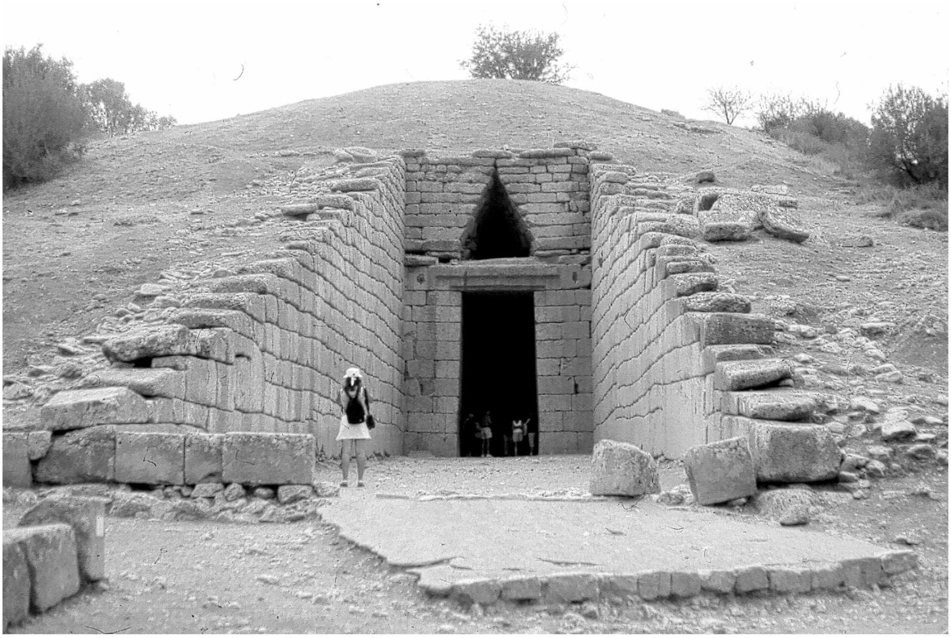

Figure 1. Tresaury of Atreus. The *dromos*, the *stomion*, and the mound

The access is through a rectilinear ramp, known as *dromos*. The ramp follows a radial direction, cutting the tumulus and the circular walls **(fig.2)**. Where the ramp meets the walls, the access portal, known as *stomion*, is created. Structurally, the doorway opens the masonry circle till the lintel's height. The ends of the stone circular walls constitute the portal's jambs; one or more stone slabs realise the architrave, at the former level of the ground, which corresponds to the upper edge of the pit. Above the lintel an opening is created throughout the whole depth; the opening is shaped as an isosceles triangle on the outer facade, following the construction device, known as relieving triangle.

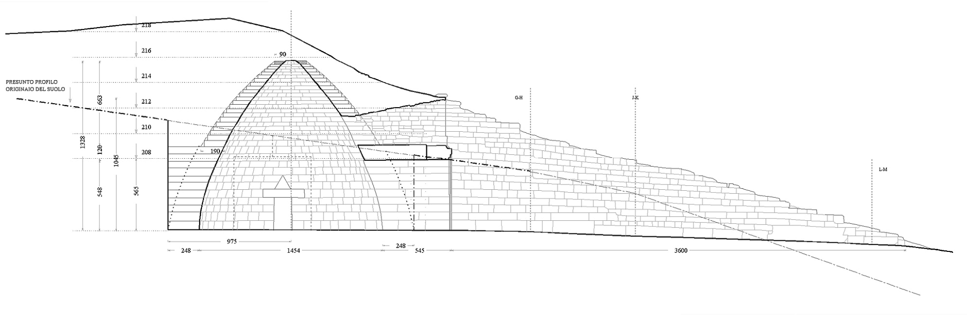

Figure 2. Treasury of Atreus. The longitudinal section shows the former ground profile, the dome's thickness, and the pit's height (after Wace 1921-23, pl.LVI and Wace 1940, Figg. 1 e 2)

The chamber walls are realized using stone elements slightly cantilevered and arranged in successively concentric rings. The stone rings gradually close the internal space, creating a pointed dome at the intrados. The dome's inner height at the keystone is proportional to the room's size, nearly corresponding to its internal diameter. Wace states (1921-23, p.290) that the height equals the diameter so that the vertical section is square inscribable. Pelon (1976, p.335) considers the height generally smaller than the diameter, often between 1/10 and 1/30 of its dimension.

The arrangement in horizontal courses of slightly cantilevered stone elements constitutes the characteristic building system of the Mycenaean *tholoi* **(fig.3)**. For this reason the *tholos* masonry dome is generally known as pseudo-dome. The definition implies a structural model of behaviour: the pseudo-dome stress field is only due to vertical loading condition. Similarly to the false arch, where the arrangement through horizontal courses doesn't realise the typical arch transmission of stresses, in the pseudo-dome each of its stone slices, which ideally constitutes the dome's meridian section, are conceived structurally independent, through the transmission of only vertical forces.

On the contrary, a careful analysis of the masonry construction shows a variety of technical devices adopted to join the stone blocks, shaping the dome masonry as a single volume. Such characteristic construction features –clearly evident from the site analysis and also recorded by various archaeologists during excavation- didn't constitute object of documentation on the masonry structure. In particular, due to the difficulty of measuring the subterranean structure, there aren't yet comprehensive surveys of one complete *tholos*, showing the arrangement of the 'hidden' parts. Only in few cases of reconstruction or due to partial collapses, it has been possible to survey the internal arrangement, the masonry thickness at various levels, and the extrados morphology, together with the relationships with the pit's surface and the mound.

Among the available documentation on the masonry arrangement of the chamber, a plan showing the upper course of the dome in the Treasury of Atreus is particularly relevant, surveyed at the beginning of the 19th century (Blouet 1833, p 66 fig.4), when the whole thickness of the stone course became visible, due to the lack of the keystone slab, later reconstructed. The survey plan shows the horizontal stone course composed of trapezoidal blocks on a single row, touching each other only at the intrados for approximately 8 cm **(fig.4)**. For the remaining parts, the connection between the blocks is realised by means of forced insertion of small stones, which –as wedges- tighten the blocks horizontally.

Figure 3. *Tholos* of Korìfasio. Detail of the dome masonry

The analysis of the masonry arrangement shows that the Mycenaean *tholos* presents features which can effect its static behaviour. These features suggest the need to verify the validity of the hypothesis of the pseudo-dome model generally attributed to such structures. The inclusion of the dome inside the excavation pit allows the retaining of the dome at the lower level. The presence of stone wedges among the blocks shows the intention of tightening the horizontal masonry rings, which constitute the parallels of the stone dome. Moreover, the offset of the vertical joints ties the structure along the meridians. All those features constitute the technical devices, which ensure unity to the dome structure, suggesting a model that contrasts to the vision of a dome of independent radial slices.

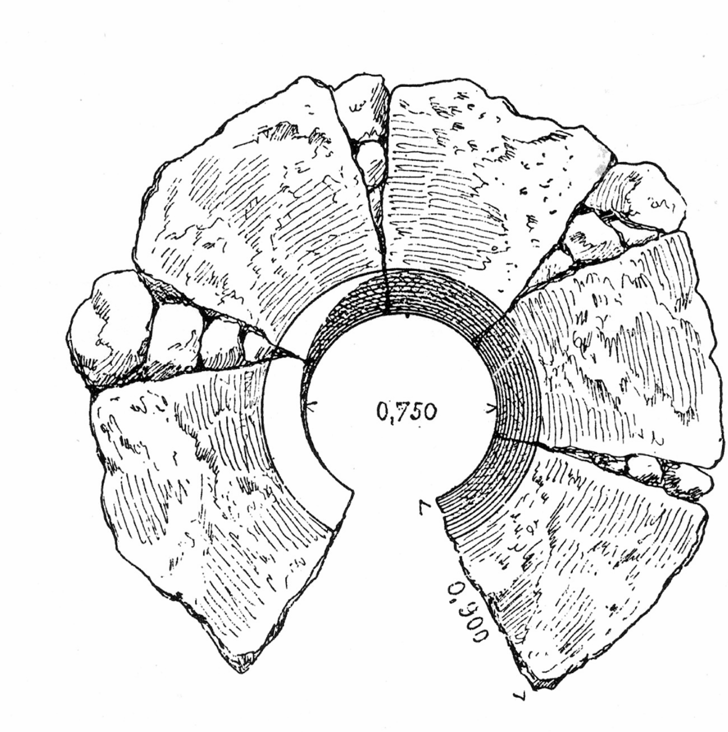

Figure 4. Treasury of Atreus. Plan of the upper masonry course in the dome (Blouet 1833)

THE DOME OF THE TREASURY OF ATREUS

Study of the actual state and construction of a geometrical model

The *thol*os hypogean condition doesn't allow the direct survey of all the structural elements located in between the chamber and the extrados of the mound. Therefore it is difficult to measure the masonry thickness, to distinguish the built parts from those obtained through rock digging, as well as to define the mound and the internal masonry arrangement. Generally the existent survey drawings represent the dome profile at the intrados and the elevation of the stone elements, leaving any data of the 'hidden' parts of the hypogean structures, necessary to further investigate the construction and the structural behaviour.

The investigation of the specific case-study of the Treasury of Atreus leads to the understanding of the complete architectural model of the *tholos*. Through the examination of specimens and surveys, existent in literature, it has been possible to formulate a realistic hypothesis on the geometric configuration of the monument.

The De Jong survey (Wace 1921-23, Tav. LVI) has been elaborated with additional data from other sources. The thickness of the upper stone course before the keystone has been obtained from the survey plan of the upper horizontal course of the dome (Blouet 1833, pl66 fig.4). In the survey plan by De Jong the depth of the access opening to the secondary chamber indicates the thickness of the masonry dome at the ground level. The profile of the excavation pit is partially traced in the transversal section, near the relieving triangle of the architrave of the doorway of access to the secondary chamber, showing the inclined wall toward the chamber. The chamber is realised inside a great cavity cut into the rock with walls slightly inclined toward the interior (Pelon 1976, p.174). It has been possible therefore to verify the thickness of the masonry walls only in three spots. The dome extrados geometry has been obtained through approximation connecting the three noted points with a circular arch. The stone courses thickness has been defined as the horizontal distances from the traced curve to the intrados survey profile, at the middle of each stone course. It has been possible therefore to differentiate in section the masonry structure of the dome from the mound above **(fig.5)**.

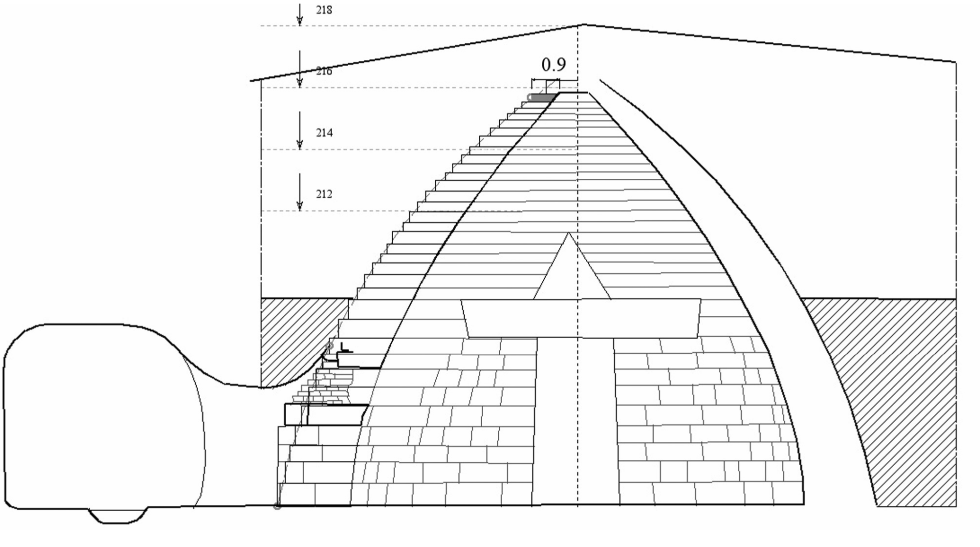

Figure 5. Elaboration of the De Jong survey. The three reference points are indicated

From the transversal sections of the *dromos* after the surveys on the retaining walls (Wace 1940, p.239, fig.1 and p.241, fig.2) the former ground level has been obtained, at the four spots where the sections were carried out. Along the longitudinal section the whole former ground profile **(fig.2)** has been obtained connecting the known points to the doorway lintel's level and prolonging this line inside the chamber. The comparison between this profile with the internal height of the secondary chamber, dug in the rock, helped obtain the height of the excavation pit, which has been approximately defined as correspondent to the level of the doorway's architrave extrados, of 6.70m from the chamber ground.

The extrados mound profile has been obtained using the information on the survey plan of the investigation realised by Wace (1956, fig.7, p.120) from 1939 to 1955 around the tomb. The survey plan shows all the excavations' trenches, the contour lines of the mound, and the chamber and *dromos* plans.

The mound materials have been presumed extending the results from the Wace surveys realised in the areas of the mound at the back of the *dromos* retaining walls (Wace 1940, p.239, fig1 and p.241, fig.2).

The final 'interpretative' drawing shows the geometric model used in the following static analysis.

STATIC ANALYSIS OF THE MASONRY DOME

Inadmissibility of the pseudo-dome static model

On the geometric model previously traced the equilibrium of a single meridian slice of the portion of the *tholos* emerging from the pit has been examined **(fig. 6)**. The angular width of the slice is 11,25° while the vertical loads considered are the weight of the masonry wall portion and the weight of the rubble stone and clay mound portion. The slice is composed by successive courses of single stone blocks of actual height. The number of the possible overturning mechanisms is 22. In place of a continuous profile at the intrados it has been considered a stepped profile. The mound above is divided into 23 volumes according to the geometrical construction of fig. 6.

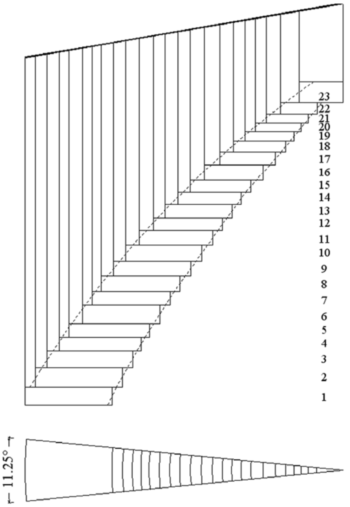

Figure 6. Section and plan of the meridian slice.

According to the geometric model, the weight of each block and the weight of the mound portion have been quantified. The assumed specific weight of the conglomerate blocks is 2,5 t/m^3 while for the mound the average specific weight is 1,9 t/m^3. The equilibrium condition of the various volumes has been examined for all the possible collapse mechanisms. A single mechanism of rotation of the slice and the above mound around the hinge A is represented in fig. 7.

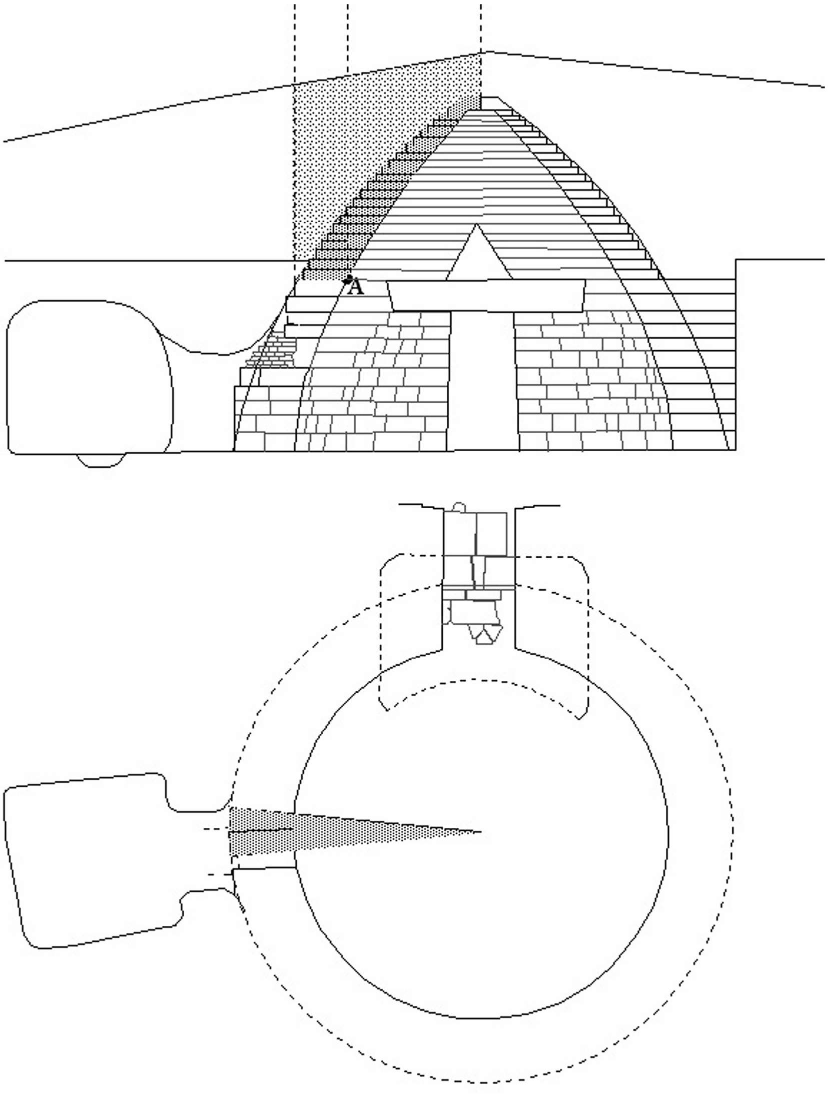

Figure 7. Section and plan of the meridian slice. The mechanism of rotation of the slice and of the above mound around the hinge A

The resisting and overturning moments have been evaluated placing the weight forces at the correspondent gravity centres. The ratios between the resisting and overturning moments define the safety factors of the considered mechanism. Table I gives the values of the stabilizing and overturning moments as well as the correspondent safety factors' ratios for the 22 collapse mechanisms. Safety factors smaller than the unity occur when the rotation hinge is placed at the points 1,2,3,4,5,6,7, and 8. The minimum value of the safety factor corresponds to the position 1. The meridian slice cannot be autonomously in equilibrium. The corbelling model of the *tholos* fails.

Table 1. Values of the stabilizing and overturning moment and safety factor for the 22 collapse mechanisms

N. mecc.	MS	MR	MS/MR
1	29,166621	43,3404872	0,67296477
2	26,3433808	37,4215096	0,7039636
3	22,6227015	31,2231469	0,72454905
4	19,8635235	26,4744291	0,75029091
5	17,6869046	22,7916323	0,77602624
6	16,7391592	19,5819574	0,85482564
7	13,8685317	15,4565578	0,89725875
8	12,4299059	12,9471507	0,96004953
9	10,8030127	10,2430254	1,0546701
10	9,30979093	7,96901093	1,16824924
11	7,68884948	6,05128962	1,27061337
12	5,89036376	4,56430373	1,29052844
13	5,00399913	3,57433318	1,39998116
14	3,91876029	2,63292584	1,48836713
15	3,08906493	1,94354593	1,58939642
16	2,26848545	1,33740651	1,69618245
17	1,73661547	0,78448415	2,2137037
18	1,30394402	0,46812007	2,78549053
19	0,95780808	0,28388068	3,37398119
20	0,69843751	0,18728004	3,72937499
21	0,47569747	0,12108027	3,92877777
22	0,23731686	0,04605386	5,15302876
23	-	-	-

The same analysis has been developed in reference to the horizontal thickness of the dome assumed by Cavanagh and Laxton and in reference to the Treasury of Atreus geometry, as well as to the actual configuration of the mound. The horizontal thickness of the upper course estimated by Cavanagh and Laxton is 38 cm ca., value that differs from the actual measure of 90 cm as stated in the survey (Blouet 1833). Table II shows the values of the resisting and overturning moments and the correspondent safety factors. Also in this case the equilibrium is impossible because the top of the *tholos* collapses for overturning around the toe at the course 16. In any case the safety factor values are only slightly bigger than the unity for all the collapse mechanisms as it occurs in the previous model. Those results clearly differ from those of Cavanagh and Laxton (1981).

Table 2. Values of the stabilizing and overturning moment and safety factor for the 22 collapse mechanisms

N. mecc.	MS	MR	MS/MR
1	61,1884716	43,0436845	1,42154354
2	50,6848645	37,1358204	1,36485108
3	42,9869205	30,9094544	1,39073695
4	36,7546571	26,2820776	1,39846848
5	30,9607209	22,482504	1,37710288
6	24,125206	19,4056048	1,24320815
7	19,796251	15,1662739	1,30528112
8	15,484543	12,5927699	1,22963757
9	11,9285297	9,89013091	1,20610433
10	9,07267675	7,66224544	1,18407545
11	6,67280406	5,73668628	1,16318093
12	5,04149306	4,26510885	1,18203151
13	3,74897989	3,27899449	1,14333217
14	2,68751973	2,33783799	1,14957484
15	1,82442623	1,69180158	1,07839256
16	1,03785322	1,11197605	0,93334134
17	0,59475318	0,57455828	1,03514857
18	0,34968485	0,2870208	1,2183258
19	0,21212664	0,14824763	1,43089389
20	0,12345351	0,06445083	1,91546813
21	0,06125566	0,02286277	2,67927591
22	-	-	-

Cavanagh and Laxton assumed the mound flat -at the level of the dome apex at the extrados-, and in this case the destabilizing effect of the weight of the mound vanishes and the corbelling model of the *tholos* becomes admissible. The actual mound presents a thickness at the apex of about 2 metres and the extrados profile slopes down towards the springers: in this case the weight of the mound realises a strong destabilizing effect. Therefore, along the corbelling principle, the structure is in equilibrium according to particular mound shape. This condition is clearly not satisfying. We can consequently assume that the corbelling model cannot constitute the static behaviour since the safety of the *tholos* dome has to be preserved for any profile of the mound.

The various slices, unable to sustain by themselves, start to overturn and lean against each others putting in compression the dome rings. The capability of the horizontal rings to sustain the radial compression is due to the well arranged horizontal courses **(fig.4)**, with the small wedge-like stones inserted into the interstices among the larger stones. This special arrangement mobilizes the mutual compressive interaction across the vertical sections of the annular rings and it avoids the overturning of the single slices. A membrane behaviour of the dome arises. The compression inside the rings, together with the weight of the masonry and the mound, produces a resultant force acting along the middle surface of the dome. As it will be shown later, this membrane behaviour is consistent across the whole dome middle surface from the apex to the basis of the excavated pit. Another construction device of the Mycenaean builders consisted in realising a compact stone which fills the gap in between the dome and the pit (Bohn 1880).

THE MEMBRANE BEHAVIOUR OF THE THOLOS

According to the typical behaviour of the masonry domes, tensile stresses arise in the rings toward the springers and the dome suffers meridian cracks. The masonry dome behaves as a sliced dome, which can be analysed using the arch equilibrium. We observe, on the other hand, that in the case of the *tholoi* the behaviour is different because the tensile stresses required for the membrane equilibrium are produced by the counter reaction of the rock strictly adherent to the wall, due to the insertion of the dome into the pit and thanks to the filling between the rock and the masonry wall.

Fig.8 shows the graphic procedure used to evaluate the horizontal forces acting on the vertical faces of the slice for each single course, due to the resulting forces in the rings. The magnitude of the resultant horizontal action, transmitted by each course on the slice, added to the weight of the stone block and to the weight of the corresponding portion of the mound, is such to produce a resultant force passing through the middle line of the slice. Hence, this resultant horizontal action is univocally determined.

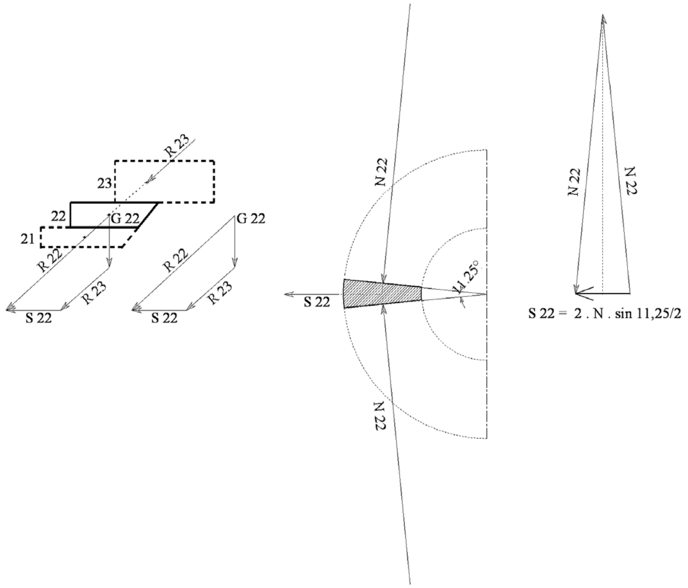

Figure 8. The graphic procedure used to evaluate the horizontal action S_{22}

Fig. 8 shows the evaluation of the horizontal action S_{22} transmitted by the course 22 on the centre of the block 22. We consider, for instance, the block 22 of the slice according to the model in figure 6. The following forces, all passing through the centre of gravity of the block 22, are so defined:

- the action R_{23} is the meridian force passing through the centres of the blocks 22 and 23, transmitted by the upper course 23 to the block 22
- G_{22} is the weight of the block 22
- S_{22} is the horizontal action transmitted by the ring course 22 to the block 22, i.e. the resultant of the horizontal forces acting on the side faces of the block 22, due to the internal force acting in the course ring 22
- the action R_{22} is the meridian force passing through the centres of the blocks 22 and 21, transmitted by the course 22 to the lower block 21

The resultant of the action R_{23}, with the weight G_{22} of the block 22 and with the force S_{22} is the force R_{22} passing through the centres of the blocks 22 and 21.

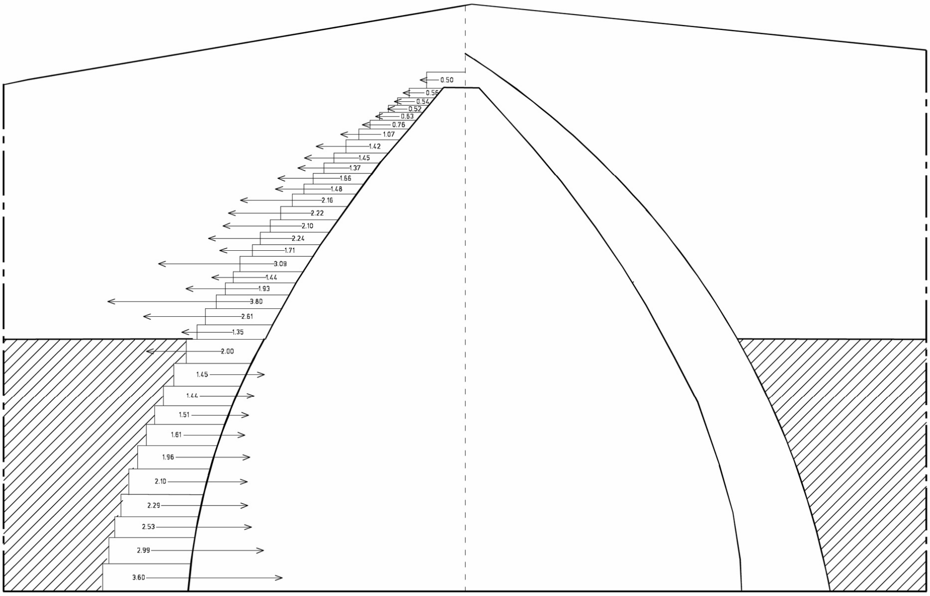

Figure 9. The horizontal actions

Fig. 9 shows the sequence of the forces S_i ($i = 1,2,...N$) if N represents the number of the courses evaluated according the previous procedure. We can notice that in the lower part of the *tholos* the forces S_i change direction. In this case these forces S_i are absorbed by the excavated pit thanks to the compact stone filling behind the wall: in these case the correspondent ring courses are unloaded.

CONCLUSIONS

The results presented here challenge the simplified model of the pseudo-dome generally considered for the Mycenaean *tholos*. Moreover, the study shows the technical and architectural devices wisely realised by the Mycenaean builders in order to ensure a great stability of the structure. Therefore, in the history of the development of construction techniques it seems that the dome construction appears before the masonry arch. Those considerations contribute to the understanding of the development of the structural knowledge which lead to the conception of the static principle of the dome and the arch.

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