

# Historical Static Analysis of the Coliseum

Simona Coccia, Mario Como, Maria L. Conforto and Ugo Ianniruberto

## STATICAL ANALYSIS OF THE AMPHITHEATRE IN ITS ORIGINAL CONFIGURATION

### Stresses in the piers

The first three elliptical rows of travertine piers, the ambulatory vaults and the attic constitute the most structurally problematic part of the Coliseum. The construction of the ambulatory vaults has been shown to have taken place in several building phases, in separate parts which only subsequently were joined, dismantling the centering at different times (Rea, Beste and Lancaster, 2002). This suggests that these vaults behave like a series of radial arches placed on the walls, joined together to form a single unit (**Fig. 1**).

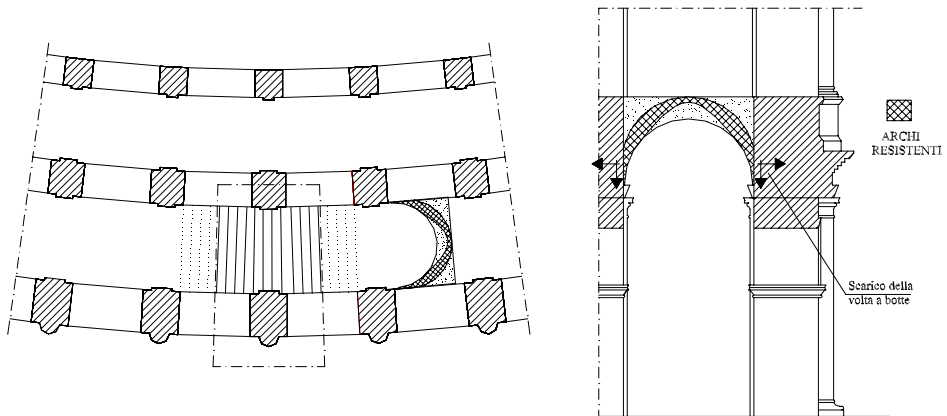


Figure 1. The resistant system of the radial arch vaults

Roman concrete, with the use of pozzolana in the preparation of the mortar, was extremely resistant, particularly in tension, and long-lasting, as is clearly visible today. Evaluating the stress to which the ambulatory vaults are subjected should necessarily take account of the tensile strength of the material. Firstly, however, the continuous presence of tensile strength cannot be admitted along the circumferential direction because of the above mentioned building process of the vaults. Secondly, it should be also observed that considerable time was required, following casting, for the *opus caementicium* to reach the necessary consistency to take this tensile strength. The casting of the concrete, needed to build the ambulatory vaults, was done using centering and scaffolding: it is extremely likely that, when the centering was dismantled, the concrete had not yet reached sufficient strength to absorb tensile stresses (Giovannoni, 1925). On the other hand, if it were

accepted that after dismantling, the concrete of the vaults did have sufficient strength in the radial direction, then the same lateral deformation of the outer wall and the inevitable little shocks caused by the many episodes of seismic activity during the time, may have caused the vaults to crack and thus lead to the loss of tensile strength. It is therefore reasonable that the calculation of the stress exerted on the ambulatory vaults must be carried out by assuming for the concrete the no tension model of material. Consequently states of minimum thrust will come into force the vaults. Indeed, the walls, and in particular the outer ring wall which receives the thrusts of the external ambulatory vault, undergoes small rotation and lateral deformation. The piers of the outer wall are therefore exposed not only to the vertical and horizontal radial actions of the ambulatory vault but also to the imperfectly aligned thrusts of the circumferential arches. The opposing thrusts of two adjacent circumferential arches do not completely cancel out one another and cause a *radial* action  $S_r$  which still tends to bend outwards the external wall. If the thrust of the arch is indicated by  $S_c$  and the curvature of the plan by  $r$  ( $r=l/R$ ), then (Fig. 2):

$$S_r = S_c \cdot r \cdot i$$

with  $i$  standing for the distance between the piers. The equivalent load, radially distributed on the external wall is given by the equation:

$$q_r = S_c \cdot r$$

In the original configuration of the amphitheatre there were no tie-beams or radial chains: the thrusts  $S_c$  or, equivalently the radial loads  $q_r$ , are exerted on the piers, in particular on those of the outer wall.

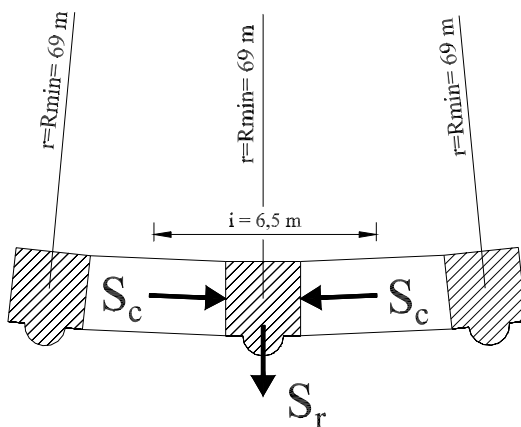


Figure 2. Radial actions  $S_r$

The ring walls can be obtained by the repetition of a single cell represented by a pier together with the adjacent ambulatory vaults, the architraves and the arches below, all taken along the piers semi-spans. The following calculations refer to this structural cell. The weight of the outer wall is calculated by considering the weight of the attic of the piers and of the parts of the circumferential arches. The calculations refer to the piers close to the main axes, where the thrusting forces are greatest. The radius of curvature of the outer wall in this area is equal to:  $R_{\min}=69$  m.

This situation relates only to the piers situated on the main axis where the thrusting forces are larger. The internal pier undergoes very low stresses because subjected to a centered axial load. The base section of the intermediate wall is also completely reactive. The external pier base section, on the contrary, is partially reactive. To properly evaluate stresses in the external pier, we should take into account that the structure of the pier is composed of large blocks positioned in an organized pattern. Nevertheless, in a first analysis which assumes a continuous no tension model for the pier, it was found that only a part of the geometrical section at the base of the semicolumn-pier is compressed and that the value of the maximum compression stress was found to be about 9.6 MPa. This value is however much lower than the crushing strength of the travertine which usually reaches at least 30 MPa.

In order to improve our understanding of this aspect, it is worth examining the behavior of this complex structural system from another perspective. Limit analysis makes in fact possible to evaluate the effective strength of the monument under the vertical loads (Lauri, 1999, Como et al., 2000, 2001).

### **Limit Analysis**

In the following section the behavior of the structures of the Coliseum will be examined from the viewpoint of limit analysis with the assumption that the material has not tensile strength. The study adopts a rigid no-tension constitutive model with no sliding and material penetration (Heyman, 1966, 1982). The possible mechanisms by which the structures of the Coliseum may be deformed should be kinematically compatible and should not therefore allow slide and material penetration. Only at the toe of the external piers, on the contrary, the finite crushing strength of the travertine has been taken into account, according a simple approach (Como and Grimaldi, 1984).

The analysis has firstly examined the possibility of *local failure*, characterized by some piers being pushed outside. It is easy to show that any local mechanism which admits that only part of the whole of the piers could rotate outside is incompatible. A kinematically compatible mechanism involves on the contrary the outer rotation of *all* the eighty piers of the three ring walls. The circumferential arches of the wall would crack, splitting into blocks and accompanying the movement, continuing to exert thrust. For the masonry domes the situation is similar. The problem

that will be examined here is to assess the magnitude of the live loads acting upon all the ambulatories capable of bringing the original structure of the Coliseum to the point of collapse.

The collapse mechanism, which involves the overturning of one or more circumferential walls, causes the expansion of the ambulatory vaults and of the ring walls. For the masonry domes the situation is similar. Figure 3a shows a spherical masonry dome subjected to its own weight and to the weight of the central lantern. The fact that the material is no tensile resistant causes meridian fracturing. (**Fig.3a**). These meridian fractures split the dome into segments which converge in the central supporting ring of the lantern. The collapse is axial-symmetric and occurs with meridian cracks accompanied by two ring cracks running along the parallels at the intrados and at the extrados of the dome (**Fig. 3b**).

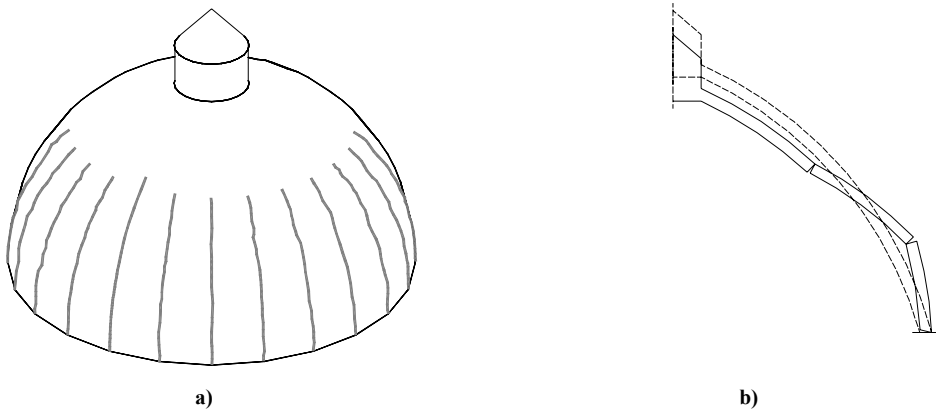


Figure 3. Masonry dome: a) Axial-symmetric view showing the meridian fractures  
b) Mechanism deformation of a segment of dome

The Coliseum possible failure mechanisms can be of two types:

- mechanism type a) involving the rotation of the whole external wall (**Fig.4a**).
- mechanism type b) and c) involving the rotation of both the whole external and central walls (**Fig.4b** and **4c**).

In the following sections the two possible mechanisms of the structure will be analyzed. The first mechanism considers only the external wall and the external ambulatory vault. The external wall rotates outwards while the inside and the central walls keep still. The ambulatory external ring vault, included between the central and the rotating external walls, cracks and three hinges will form transversally, two at the intrados of the abutments and one at the extrados. Figure 4a shows a possible mechanism of the structure and indicates an arbitrary distribution of hinges.

The force system acting on a slice of the monument is constituted by the following components:

- the weight  $W$  of the walls;
- the dead load  $p$  acting on the ambulatory vaults;
- the live load  $\lambda q$  acting on the ambulatory vaults;
- the thrust  $S_c$  dependent on the dead and live load

By applying the kinematical theorem the function  $lq$  is given by:

$$\lambda q = \frac{L_{dead}}{L_{live}}$$

The minimum of the function  $\lambda q (d, d_i, x_i; i=1..N)$ , calculated with the constraint expressing the equilibrium of the wall in the vertical direction at a fixed level of the masonry compression strength, provides the collapse load  $\lambda_c q$  related to the set of the analyzed mechanisms. The minimum of the function  $\lambda q$  among all the various mechanisms sketch in (Figs 4a, b and c) gives the collapse load  $\lambda_c q = 11.47 \text{ kN/m}^2$ : it corresponds to the mechanism type a) (Coccia, 2001). The application of the vertical loads on a reduced number of ring vaults gives a higher collapse load (Fig. 5).

The magnitude of the collapse load seems to be rather low and confirms the suspect that the external ring wall has tendency to rotate outside because the vault thrusts, in spite of the presence of the inner offsets in the wall, that tend to bend the wall on the inside. The lack of radial constraints for the ring walls, as stone lintels - on the contrary visible in other Roman amphitheatres, as for instance in the amphitheatre of Nimes - conditions the static safety of the monument in its original form under the vertical live loads.

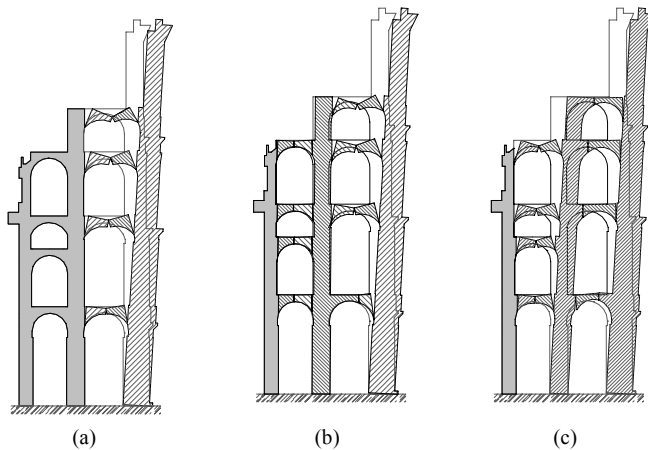


Figure 4. Radial section of the mechanism

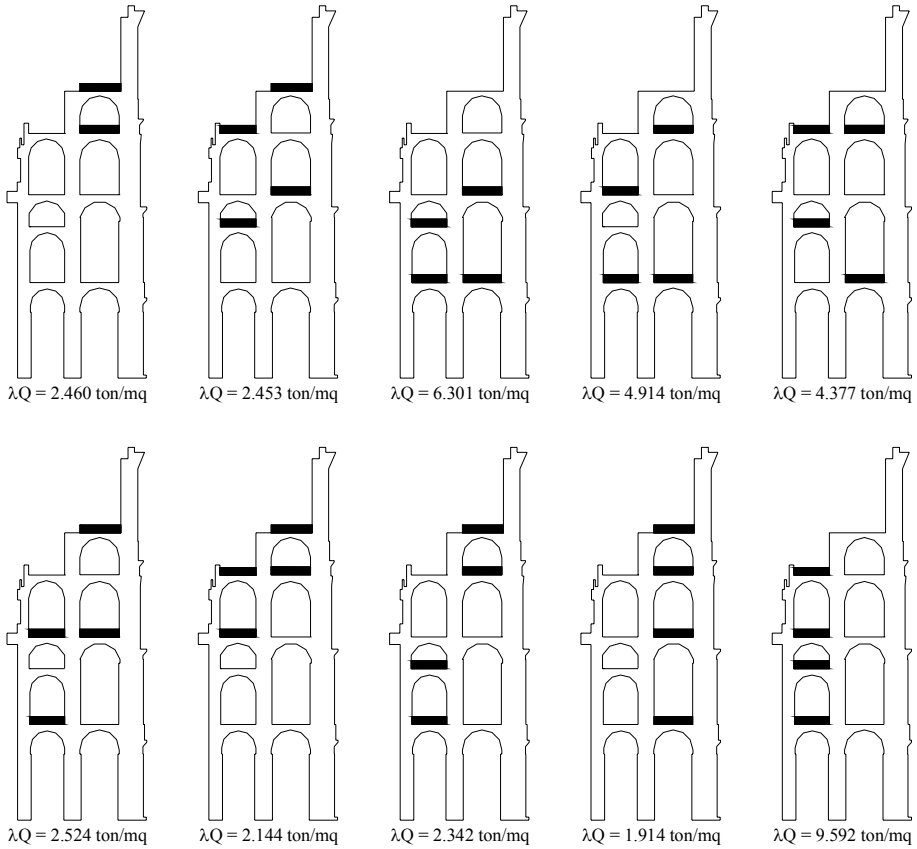


Figure 5. Various combinations of vertical loads applied on the ring vaults and the corresponding collapse loads

### THE DAMAGE AND THE SUBSEQUENT REPAIRS

The damaged configuration of the monument in the eighteenth century is the result of nearly 17 centuries of the building's existence, during which it has alternated between periods of use and periods of abandonment during which there have been exceptional events, such as fires or earthquakes which have probably led to partial collapses, as well as to systematic demolition for obtaining construction material (Conforto, 1986, Lancaster, 1998, Croci, 1990, Coarelli et al., 1999).

Figure 6 shows the plan of the first level of the amphitheatre in its current state. The entire southern part of the monument, both the external and intermediate wall with all the corresponding piers, had been destroyed.. The outside of the arena is still well preserved around three quarters of its circumference: 50% of the outer wall is missing on the southern side, and part of the interior wall

has been rebuilt. It is unclear how the damage of the Coliseum evolved over time and how it reached the state shown in (Fig 6).

No work was done until under Pope Pius VII (1800-1823) and later Leo XII (1823-1829), Gregory XVI (1831-1846) and Pius IX (1846-1878) a lengthy process of restoration was carried out which regarded the entire amphitheatre.



Figure 6. Plan of the first level of the monument in its current state, (reprinted from Rea, 2002)

There was fear for the stability of the external wall since only the 39 arches overlooking the Esquiline hill were still left standing and the balance of thrusts and counterthrusts in the circumferential direction had been interrupted. Stern, Palazzi and Camporesi constructed the buttress towards the east, which was completed in 1807, while Valadier built the buttress on the west side. Despite the works undertaken by Stern (1806 – 1820) and Valadier (1823 – 1826), the external wall continued to rotate outwards. Therefore later, in the 1850, Canina installed a triple order of chains corresponding to the thirteen central arcades (Fig. 7).

To anchor the radial chains in the uppermost part of the external wall, 13 piers of the third level were reconstructed and 13 pairs of chains were installed in the upper floor of the second level. Once the anchoring wall structure had been completed, the chains were positioned in pairs on the same level of the vaults. The chains placed on the floor of the third level, which were about 16 meters in length, and made of two pieces joined by hooks, were linked in pairs by horizontal iron bars passing through the eyes of the chains themselves.

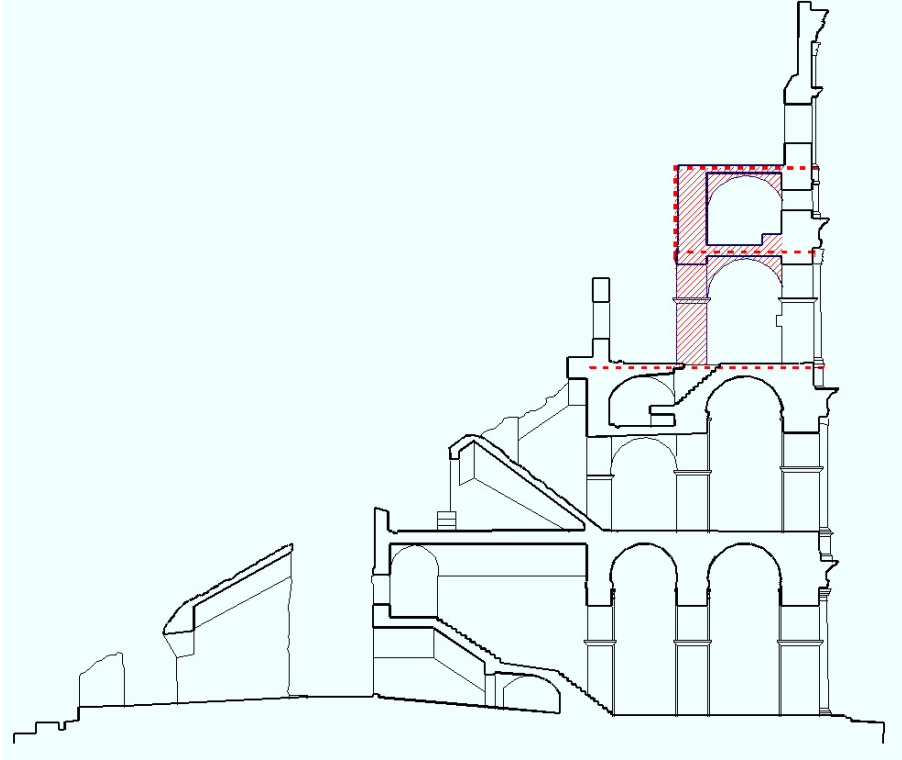


Figure 7. The chains placed by Canina at the various levels together with the new piers

The same type of iron supporting bars created the anchorage, both on the external facade as well as on the intermediate pier using iron wedges and buffers. The overlying chains, formed by a single piece of chain 9 meters long, were linked in pairs by means of a vertical bar. Canina's work completed the consolidation of the Coliseum. The whole central zone of the wall towards North-East was stabilized for the first time and the effectiveness of this work can still be appreciated today, 150 years after it was carried out. While the buttresses built by Stern and Valadier restore the balance of the thrusts of the circumferential arches, the chains installed by Canina restrain the external wall of the amphitheatre in the radial direction.

#### **POSSIBLE CAUSES OF DAMAGE: PAST EARTHQUAKES OR DECONSTRUCTION?**

Causes of the damage suffered by the Coliseum during the centuries are still unknown. There is no evidence of foundation differential settlements. Hence two different conjectures can be made: responsible of the damage are the past earthquakes (Croci and Viskovitch, 1993, Cerone et al., 2000) or, otherwise, the past deconstruction works (Conforto and Rea, 1993, Como et al., 2005).



### Seismic excitability of the monument: the effect of the soil-structure interaction

Seismic activity in Rome is quite moderate. Nevertheless, during more than 2500 years of history, the city has been interested by a considerable number of events, causing quite severe damage to its artistic patrimony. According to the Italian earthquake catalogue, spanning more than two thousand years, the major effects are exclusively due to seismic activity in the Apennines and reached in some cases felt intensities up to VII – VIII degree of the MCS scale. (Molin and Guidoboni, 1989, Guidoboni, 1994). The predominant role in the damage distribution is played by the local geological conditions: the damage is more severe for the monuments founded over the sedimentary fillings of the River Tiber. As actually observed, during an earthquake, the surface ground motion in a sedimentary-filled valley may be significantly amplified and prolonged (Moczo et al., 1995).

The soil on which the Coliseum is built, not yet completely known, in substance is formed at its base by stiff marine clays, gravels and tuffs and by a cover of soft alluvial sediments of a former tributary of the river Tiber. Only a portion of the northern part of the monument seems to be underlain by the stiffer middle Pleistocene continental deposits (Moccheggiani and Carpano, 1997, Jappelli, 2000, Funicello et al., 1995, 2002, Sciotti, 2004). It is thus reasonable to attribute to the foundation soil of the Coliseum the same maximum intensity level of the seismic motion registered in Rome. To the VII-VIII MCS degree corresponds the range of  $200 \text{ mm/sec}^2 - 400 \text{ mm/sec}^2$  for the horizontal seismic ground acceleration. This is the maximum ground acceleration level that will be considered in examining the effects of the earthquakes to the Coliseum structure.

A simple model of the structure, together with the assumed elastic connections to the underlying soil, is sketched in (Fig. 8). A rigid large block, representative of the monument, of mass  $m$  and moment of inertia  $I_o$  about the horizontal transversal axis passing through its gravity centre  $G$ , can displace horizontally and can rotate with respect to the rigid frame  $Oxy$ , integral with the soil. The system, with respect to the soil, has two degrees of freedom, the horizontal displacement and the rocking rotation. The monument cannot be considered rigidly attached to the soil but elastically connected by the translational and rotational elastic springs of suitable stiffnesses.

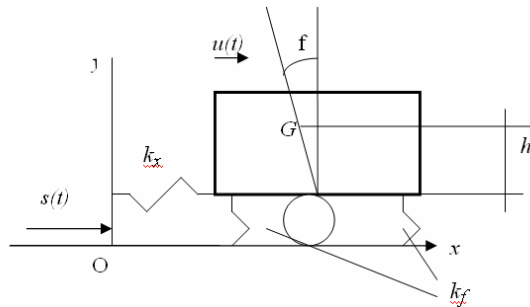


Figure 8 The simple model of the soil-structure interaction

Because of the seismic action, the soil under the foundation to which the block is elastically connected, is subjected to the horizontal motion. In a previous paper (Coccia et al., 2005) we have shown that the large mass of the building, together with the presence of soft soil, produce relevant soil-structure interaction. Consequently, the intensity of the horizontal forces imparted to the structure is strongly reduced and the seismic shear waves that propagate from the bedrock to the surface, while gradually grow up crossing over softer soils, at the surface, mitigate their action on the monument. Only small quantities of energy can be transmitted from the ground motion to the monument. At the same time, the stiff foundation structure protects the monument from the differential distorting settlements, certainly occurring in presence of soft soils.

The first oscillation mode, of very high period, is practically purely translational. The second mode, of lower period, is quasi-translational. The seismic input is a dragging horizontal motion thus mainly activates only the first oscillation mode. By using, the common spectrum response equation given by the Italian seismic Code given by:

$$R(T) = 0.862/T^{2/3}$$

we obtain  $R(4\text{sec}) = 0.34$ . Thus, while the maximum possible ground acceleration is equal to  $400 \text{ mm/sec}^2$ , the maximum acceleration that can effectively reach the monument is about equal only to  $0.4 \times 0.34 = 0.136 \text{ m/sec}^2$ , i.e. of the order of magnitude of  $0.01g$ , ( $g$  is the gravity acceleration).

### **The seismic strength of the monument**

The seismic strength level of the Coliseum structure can be roughly evaluated by means of a *push-over* analysis. The presence of a predominant translational mode suggests that the horizontal accelerations can be assumed constant across the masses of the monument. A gradually increasing distribution of horizontal loads, linearly proportional to the masses will thus represent the push-over loading condition (**Fig. 9**). One half of the entire ring vault, that the horizontal seismic action pulls and forces to expand outside, comes off the other half, which, on the contrary, as an arched dam, is compressed by horizontal pushing loads. Only one half of the monument thus can fail under the earthquakes.

By considering in plan the half of the whole ring vault pulled by the seismic loads, at a first insight we could come to the conclusion that the collapse of this vault can effectively occur with the in plane four hinges mechanism of (**Fig. 10**). This deformation of the vault, on the other hand, has to be consistent with the assumptions of no sliding and of no masonry penetration. Problems arise when the other parts of the building are considered. Beneath the ring vault there are the piers and the circumferential arches. Each pier must undergo movements in the two orthogonal directions in plan to rigidly follow the displacements of the ambulatory vaults. A plane translation of the head of a pier corresponds to its lifting.



Figure 9 The plan of the Coliseum and the direction of the seismic action

Consequently, the pier rising cannot be always the same and the vaults must distort with the possible occurrence of masonry penetration. In the context of the simple model of the rigid in compression and no tension material, compatible displacement mechanisms of the ring walls could thus not exist and the collapse load cannot be obtained by means of the classical limit analysis applied to masonry structures.

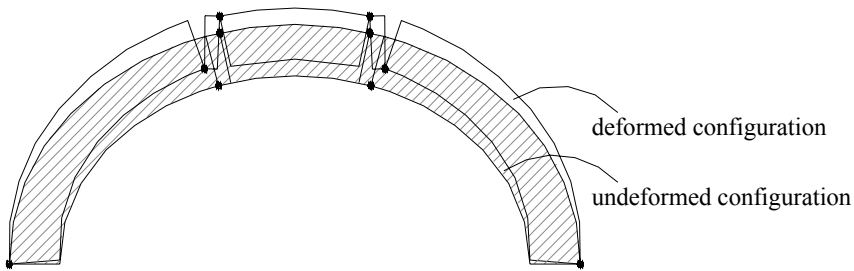


Figure 10. In plan kinematical deformation of half ambulatory ring vault and different displacements of the piers

The evaluation of the horizontal seismic strength of the monument has required the use of special non linear programs, particularly the DIANA (Diana User's Manual 1990), a finite element program of structural analysis able to solve a wide class of non linear problems, and that is particularly useful in presence of materials with different tension/compression behaviour.

As for as the input data are concerned, we have assumed an uniaxial stress-strain ( $\sigma - \epsilon$ ) relationship with vanishing tensile strength either for the piers built with travertine voussoirs or for the concrete of the vaults and the upper cornice. The crushing strength both of the travertine and of the concrete,

has been assumed equal to 2500 N/cm<sup>2</sup>. At conclusion the assumed material parameters are respectively (**Fig. 11**):

$$\begin{aligned}
 f_t &= 0.3 \text{ N/cm}^2; \\
 G_t &= 0.00045 \text{ Ncm/cm}^2; \\
 f_c &= 2500 \text{ N/cm}^2; \\
 G_c &= 400 \text{ Ncm/cm}^2; \\
 E &= 2 \times 10^6 \text{ N/cm}^2; \\
 \nu &= 0.2,
 \end{aligned}$$

where  $f_t$  and  $G_t$  respectively are the tensile strength and the fracture energy of the single mesh element to be dissipated by the single mesh element. Likewise,  $f_c$  and  $G_c$  are the corresponding values for the compression.

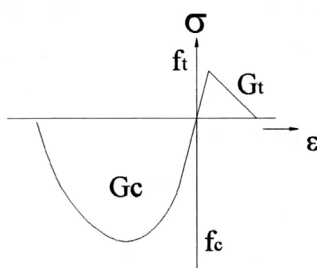


Figure 11. The assumed uniaxial  $\sigma - \epsilon$  diagram

While the vertical dead load is kept constant, the push-over horizontal loads are gradually increased as far as the snapping failure is not reached. At each step of loading it has been evaluated the corresponding horizontal displacement occurring at the top of the monument and the force-displacement diagram of (**Fig. 12**) has been plotted. Inspection of this diagram allows estimating the collapse load: it occurs at the horizontal tangent point. The loading parameter  $l$ , the ratio between collapse horizontal force and the weight, reaches the value  $l_0=0.122$ , corresponding to the action of a maximum horizontal acceleration of about 0.1g, i.e. about ten times larger than the maximum ground acceleration that could have reached the monument during its whole history.

Some of the results obtained (Marasca 2005) are reported in the following (**Fig. 13**) that shows a view of the horizontal deformations of the ring walls at the collapse.

The order of magnitude of the horizontal strength of the monument results to be much greater than the possible maximum level of the seismic forces that could have reached the monument during its whole history. The situation changes substantially if the monument is in an already damaged configuration. This consideration can explain the partial failures occurred during the past earthquakes.

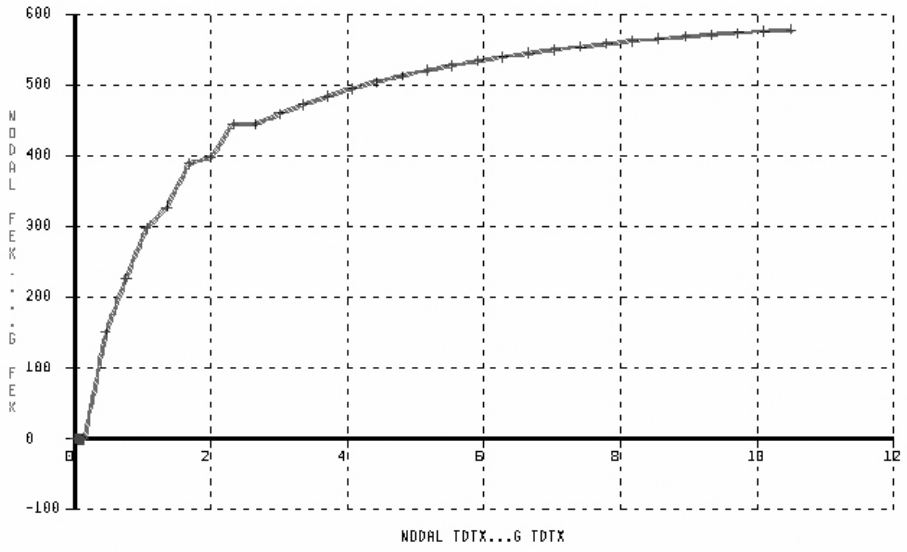


Figure 12. The force-displacement diagram

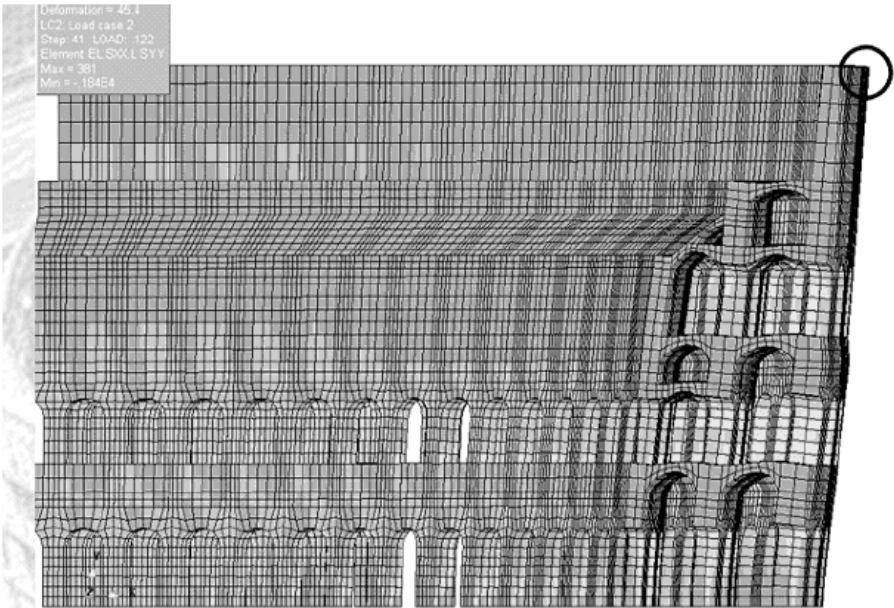


Figure 13. The horizontal deformation of the monument at the seismic collapse (Marasca, 2005)

## CONJECTURES CONCERNING DECONSTRUCTION

The deconstruction of large parts of the Coliseum was not an easy operation. Probably the demolition was carried out by provoking intentional failures, such as burning the inside of some of the travertine piers. The presence of lime workers inside the monument, who burnt marble and travertine to produce lime, may indicate the use of the fire to produce breakage of some piers and the consequent partial failures of the monument.

The demolition of one or two piers has been simulated using the program DIANA. The weight is gradually applied to the structure deprived by the demolished piers at the first level.

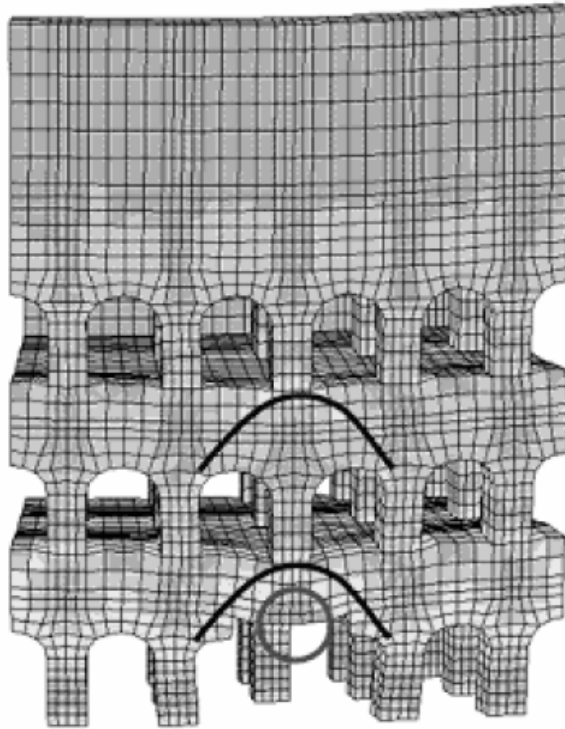


Figure 14. The arching effect in the external wall, in consequence of the demolition of a single external pier

The Figure 14 shows the effect of the demolition of a single external pier. The destruction of a single pier does not produce the collapse. The monument still stands up after the demolition of the pier because the arching effect occurring across the demolished pier (**Fig. 14**). The demolition of two piers, on the contrary, produces the collapse of all the part of the outer ring wall rising above the two demolished piers (**Fig. 15**). Figure 16 shows the step-by step weight-displacement diagram

corresponding to the gradual increasing of the weight in the two analysed cases. In the first diagram (Fig. 16a), corresponding to the cases of demolition of a single pier, the increase of vertical load with the displacement is practically linear just at the end of the loading, i.e. when the vertical load reaches the full weight of the monument. On the contrary, the load-displacement curve corresponding to the demolition of two adjacent external piers (Fig. 16b) reaches a maximum when the vertical load is only equal the 84% of the total weight.

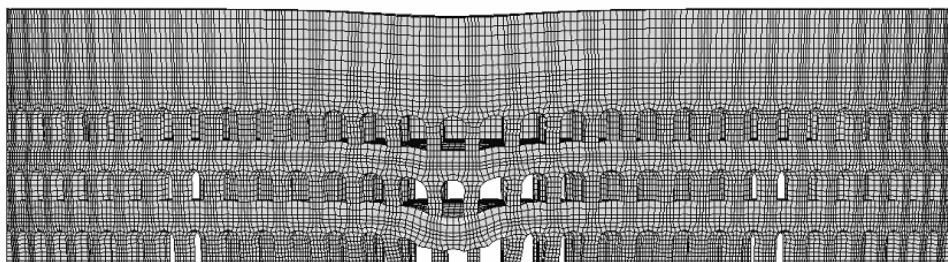


Figure 15. The partial failure of the outer wall as consequence of the demolition of two piers

Deconstruction by destroying piers therefore requires the simultaneous demolition of at least two adjacent external piers. This operation could have been carried out drilling holes in some close piers, by positioning colophony inside and igniting it by fire. Documents record various deconstruction works since the VIth century (Rea 2002).

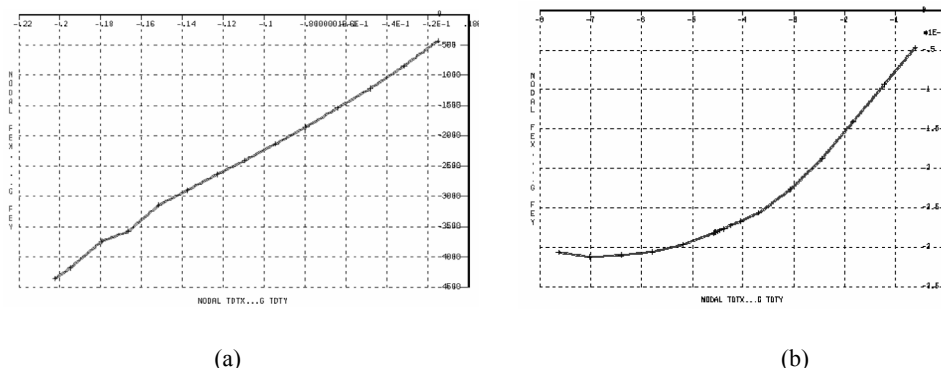


Figure 16. The step-by step weight-displacement diagram corresponding to a gradual increasing of the weight after the demolition of one (a) or two (b) external piers

## CONCLUSIONS

The application of limit analysis to the original structure of the Coliseum has shown that the external wall has a certain tendency to rotate outside because the effect of the thrusts of the ring

vaults. The introduction of radial chains, performed by Salvi, has improved the safety of the monument. As far as the reasons of the actual damage are concerned, we can conclude that the damage to the monument during nearly seventeen centuries cannot be mainly attributed to earthquakes. Because of the large mass of the building, the thick and stiff foundation structure, and the presence of soft foundation soil, the monument is, in fact, not greatly affected by seismic activity. On the contrary, severe damage has been produced by the intentional deconstruction of the monument, as nearly as the sixth century, probably carried out, step by step, by the destruction of piers at the first level of the external wall. As soon as the annular continuity is broken, owing to the deconstruction that produced outward failures of the external wall, the intrinsic weakness of the monument, due to the lack of connection between radial and circumferential walls, takes effect immediately. The external circumferential wall, once interrupted, becomes extremely vulnerable. Hence, after the first deconstruction, earthquakes acting on a structure already weakened by previous partial demolitions, can produce further damage.

## AKNOWLEDGEMENTS

The Authors thank the Sovrintendenza Archeologica of Rome for the financial support.

## REFERENCES

- Bozzano F, 1995. "Il sottosuolo dell'area dell'Anfiteatro Flavio in Roma", *Geol.Appl. e Idrog.*, 30
- Cerone M. et al., 2000. "Analisi e documentazione dei dissesti strutturali ed individuazione delle situazioni a rischio", *Quaderni del Colosseo*, 1, Sovrintendenza Archeologica di Roma
- Coccia, S. 2001. "Analisi dei dissesti delle strutture del Colosseo", *Tesi di Laurea*, Università di Roma Tor Vergata, Fac. di Ingegneria, Dip.to Ing. Civile, Roma
- Como, M and Lanni, G, 1979. *Elementi di costruzioni antisismiche*, Roma: Cremonese
- Como, M. and Grimaldi, A, 1984. "Analisi Limite di pareti murarie sotto spinta", *Atti Dipartimento di Ingegneria Civile*, Università di Roma Tor Vergata
- Como, M, Lanni, G, 1979. *Elementi di costruzioni antisismiche*, Roma: Cremonese
- Como, M., Ianniruberto U., Imbimbo M., Lauri F. 2000. Analisi limite delle strutture del Colosseo, *Quaderni del Colosseo N.1*, Sovrintendenza Archeologica di Roma
- Como, M., Ianniruberto U., Imbimbo M., Lauri F. 2001. "Limit analysis of the external wall of Colosseum" in *Proc. International Millennium Congress, More than Two Thousand Years in the*



*History of Architecture*, selected papers, vol. I: session 1 and 2, p. 1b 11, UNESCO-ICOMOS in partnership with the Bethlehem 2000 Project Authority

Coccia, S., Conforto, M.L., Como, M., Ianniruberto, U., 2005. “ On the reasons of the Coliseum structural damagement ” Intern. Seminar *Theory and Practice of construction: knowledge, means, models*, Ravenna, 27-29 Oct. 2005

Conforto, M. L., 1986. “L’Anfiteatro Flavio: costruzione ricostruzione e restauri”, *Metamorfosi* n.3

Conforto, M.L., Rea R., 1993 “ *Colosseo: alcune considerazioni tecniche* ”, Atti del Convegno ARCO, “Manutenzione e recupero nella città storica”, Roma

Croci G, 1990. “ *Studi e ricerche sul Colosseo* ”, Sovrintendenza Archeologica di Roma

Croci, G, Viskovic, A, 1993. “ Causes of the failures of Coliseum over the centuries and evaluation of the safety levels”, Proc. Of the IASS-MSU Intern. Symp., Istambul, Turkiye

Diana User’s Manual. 1990. *Nonlinear Analysis*. Frits C. de Witte and Wijtze Pieter Kikstra, Delft Netherlands

Funciello R. et al, 1995. “Seismic Damage and Geological Heterogeneity in Rome’s Coliseum Area: are they related?”, *Annali di Geofisica*, Vol. XXXVIII, n°5, 6

Funciello R., et al, 2002. “La geologia della valle dell’Anfiteatro” in *Rota Colisei*, Electa, Roma

Giovannoni, G, 1925. *La tecnica della costruzione presso i romani*, ristampa del 1994. Roma

Guidoboni E, 1994 *Catalogue of ancient earthquakes in the Mediterranean Area up to the 10th century* -I.N.G., Roma

Hall, J.R, 1967. “ Coupled rocking and sliding Oscillations of rigid circular footings” *Proc. of Int.Symp. on Wave propagation and dynamic properties of earth materials*, Univ.of New Mexico Press, Albuquerque

Heyman, J, 1966. “The stone skeleton”, *Int. Journ. Solid Struct.*, 2, 249

Heyman, J, 1982. “*The masonry arch*”, Hellis Horwood Limited Publ. Chichester, 1982

Lancaster L.C, 1998.” Reconstruction the restorations of the Colosseum after the fire of 217”, *Journal of Roman Archaeology*, vol. 11, 1998

Jappelli R. et al, 2000. “ Restauro del Colosseo”, *Quaderni del Colosseo*, 1, Area Geotecnica, Sovrintendenza Archeologica di Roma

Lauri F, 1999. “ Analisi Limite delle strutture del Colosseo ”, *Tesi di Laurea*, Università di Roma Tor Vergata, Fac. di Ingegneria, Dip.to Ing. Civile, Roma

Marasca M, 2005. “Problemi statici nelle strutture del Colosseo” Tesi di laurea, Università di Roma Tor Vergata, Fac. di Ingegneria, Dip.to Ing. Civile, Roma

Moccheggiani Carpano, C, 1977. “Nuovi dati sulle fondazioni dell’Anfiteatro Flavio”, *Antiqua*, 7

Moczo P., et al, 1995 “ Seismic response of the geologic structure underlying the roman Colosseum and a 2-D resonance of a sediment valley”, *Annali di Geofisica*, Vol. XXXVIII, n°5, 6

Molin, D and Guidoboni E, 1989. “Effetto fonti, effetto monumenti a Roma: I terremoti dall’antichità a oggi, in *Mediterranea. Storia – Archeologia- Sismologia*, edited by E. Guidoboni, SGA, Bologna, Italy

Rea R., Beste H.J., Lancaster L.C. 2002, “ Il cantiere del Colosseo”, *Boll. Istituto Arch. Germanico, Sez. Romana*, Vol. 109, Verlag P.Von Zabern. Mainz Am Rheim

Sciotti M., 2004 “Ricostruzione schematica dei terreni di fondazione del Colosseo“. *Da il Colosseo Electa*, Roma (in preparation)