Construction Techniques in Medieval Cairo: the Domes of Mamluk Mausolea (1250 A.D.-1517A.D.)

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INTRODUCTION

Medieval and Renaissance Cairo is significant in the evolution of dome construction. With a tradition of masonry construction dating from ancient Egypt, which later witnessed the Islamic practice of covering holy spaces with domes by the eighth century, Cairo contains some of the highest examples of expertise in masonry architecture. The thin domes considered in this paper are spectacular for their structural daring, their complex carving, and their speed of construction.

In the crowded urban context of Cairo, more than 200 domes still stand, indicating the presence of the shrine of a powerful ruler. This paper discusses domes that belong to the Mamluk rule in Cairo (1250-1517), a period that witnessed an exceptional production and experimentation in the construction of funerary and charitable complexes. In particular, we focus on three case studies: the domes of the funerary complexes of Umm Sultan Sha'ban (1369), Sultan Farag Ibn Barquq (1398-1411), and Amir Khayer Bek (1502), **(Table 1)**.

HISTORICAL BACKGROUND

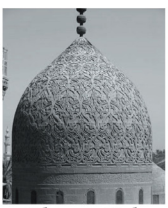
We aim to reconstruct some of the building features, the construction methods, and the structural changes of different stages of dome construction from the beginning to the end of the Mamluk rule when the characteristic shape of Mamluk domes was lost. The resources upon which this research is based are bibliographic sources, photographic material, restoration reports, and field surveys, all of which were integral in our observations and hypotheses on the history of these structures. We compared data from these resources with the result of our structural analyses, our main contribution to the field of engineering in historic structures.

Several historical questions arose during the course of this research:

- What was the state of art in dome construction in Mamluk times, and was it consistently applied or were technologic solutions based on the experience and on the innovation of a single craftsman?

- Is it reasonable to see a linear, evolutionary path for Cairene domes since they and their drums become taller and thinner in section through time?

Table 1. Comparative table of the general dimensions for the three stone carved domes of Umm Sultan Sha'ban, Farag Ibn Barquq and Amir Khayer Bek

	Total height of the dome including the drum	Internal diameter	Section at the drum	Section at the top
 Umm Sultan Sha'Ban	6.60 m	5.30m	0.33m	0.75m
 Farag Ibn Barquq	14.77m	14.43m	0.33m	0.94m
 Khayer Bek	10.12m	7.46 m	0.44m	1.08m

Through the collection of the existing available materials on these domes, and structural analysis conducted on dome sections based on our fieldwork during the restoration campaigns, we have found common features in the methods of construction and a general common understanding in the structural equilibrium of these domes. However, there are also important differences in dome technology that indicate the ability to respond to common problems, and adopt unique solutions from a dome to another.

Therefore we conclude that a margin of innovation was left to the craftsman to address issues in the construction and carving of these domes that likely differed case to case; given the number of domes (around double the 200 that remain today) that were produced in only 250 years and the speed of their assemblage, (we know from documents that the construction process for the whole

Mausoleum was usually not more than one to two years), that construction sites were active, and we can assume there was a fairly consistent exchange of information and possibly of workers among them.

Concerning the evolutionary path of Cairene domes, the study of numerous examples shows a progressive understanding of the structural equilibrium that is firmly tied to an increasing will to reach greater heights and more complex stone carving. However, progress is not strictly linear; at the beginning of the 16th century, when we witness the most stunning example of elongated carved domes, brick domes were still present and did not differ extensively from those of the previous centuries. The same fate applies to the decoration pattern, although from the 13th to the 16th century there is increasing sophistication in the development of carving motifs though an earlier zigzag model can still be applied in the later period.

The Mamluk domes are daring structures when compared to other masonry domes; they are defined simply by a number of recurrent elements: a round plan, a section referring back the tradition of various Islamic arches, and experimental patterns of ornamentation based on repetitive motifs. However, these minimum requirements allowed builders to push the boundaries of the static equilibrium. The domes are very thin and are supported on a high drum.

No written sources or drawings describing the construction process of Mamluk complexes are known to exist; the only account available comes from *waqf*, sophisticated endowment documents written at the time mainly to guarantee the ownership and the maintenance of the building. Therefore, we can only construct hypotheses on the construction of these buildings.

We begin with in-depth studies of these Cairene domes in which we compare their geometries, construction materials and decoration patterns. We have also analysed the structures of three significant stone Mamluk domes using existing and new methods in order to discuss their stability and whether centring was required for their construction. By combining the tools of engineers and historians, this interdisciplinary approach lends support for hypotheses made, and eventual conclusions about the execution of these structures.

We have looked at 113 domes of which some documentation was available, and produced a catalogue showing their construction dates, locations, profiles, and functions of the buildings that they cover. Although this paper includes images of only three stone dome case studies, we have researched the largest number of Cairene domes to develop our findings about innovations, revivals in geometries, materials, construction techniques, and decorative solutions in Mamluk times.

A large portion of the study is based on the nineteenth century work by K.A.C. Creswell who provided a fundamental contribution in the field of Islamic architecture by offering an extensive collection of accurate surveys of building complexes, architectural drawings, and an invaluable photography archive (Creswell 1959). Kessler, who in the 1970s published *The Carved Masonry Domes of Medieval Cairo*, followed Creswell's method of cataloguing and searching for an evolutionary path in the Cairene domes (Kessler 1976). The present work considers the data and the observations made by these two scholars and aims to add other tools of investigations such as structural analysis and restoration reports for an understanding of changes in the Mamluk domes.

An important transition in the history of dome construction was the shift in material from the use of brick, which dated back to the Fatimid rule, to the use of stone blocks during the first half of the fourteenth century. Around 1322, stone ribbed and carved domes began to prevail and the percentage of brick domes in new construction dropped down to only 20%, demonstrating the growing preference for stone material. Kessler analyzed the different periods of domes construction. She recorded the passage from exposed brick to plastered brick to stone carved domes. In particular, she underlined an evolution in the treatment of the joints that allows the decoration patterns of several generations of carved stone domes to become not only more complicated but also increasingly congruent with the underlying grid of stone ashlars. We collected her findings in a layout that reveals the correlation between the historic period and the development of a different style in the dome construction and carving **(Table 2)**.

Building on observations by Creswell and Kessler, we further studied and compared the change in dimensions and proportion of the domes constructed in the two materials. The average dimensions of brick domes usually range between 4 and 7 m in diameter and between 4 and 7.5 m in height. After the shift to stone materials, the dimensions for stone domes generally increased to between 8 and 10 m in diameter, and between 7 and 11 m in height. Interestingly, the overall proportions of the domes remained relatively consistent. Exceptions to these generalizations occur as in the case of one of our case studies, the Mausoleum of Farag Ibn Barquq, which has a 16 m diameter and a height of 12.8 m. However, the thickness of the stone domes in section changes significantly; while most brick domes are between 40 and 50 cm thick in section, the thickness for stone structures ranges from 32 to 43 cm.

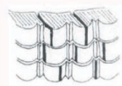
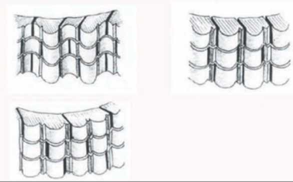
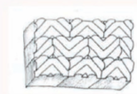
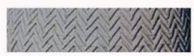
We have inferred the following general characteristics in the geometries of Mamluk domes:

- In general, the Cairene stone domes are single shelled with few exceptions like in Al-Sultanyya Madrasa, 1360 A.D., which has a double dome, very similar in shape to the one of the Timurid Gur Emir complex in Samarcanda, 1403 A.D, although in the first case the construction is in stone and in the second it is in brick.

- They follow a method of construction in which the direction of the stone ashlars is normal to the generating curve of the dome surface. The ashlars are assembled in circular rings, and decrease in size towards the top **(Fig. 1)**.

▪ The internal and external dome profiles of these pointed domes utilize two centres, unlike the single centre of a hemispherical dome.

Table 2. Classification for different periods in the master of dome construction and decoration (Kessler 1976), (Cipriani 2005)

Previous use of brick in the construction of domes, construction details	
Use of stone in the construction of domes **I -Stage of inexperience and imitation of brick decoration** Shaykh Ahmead Al Qasid, 735/1335 Amir Qaraqoga Al Hasani, before 853/1449 Khawand Tatr al-Higaziyya, 761/1360	
II- Structure in conflict with decoration Amir Tankizbugha , c.760/1350 Amir Tankizbugha , Rabi' c.764/1362 Sultan Shaban, 770/1368-9 Amir Yagay al-Yusufi, 774/1373 Al-Sultaniyya 8th/14th c. Amir Yunis al Dawadar after 783/1381	
III -Decoration and structure first properly coordinated Amir Mahmud AL-Kurdi, 797/1394-5 Sultan Farag ibn Barquq, 803-10/1400-7 (Northern Mausoleum), Amir Ganibak, at madrasa, 830/1426-7 (zig zag decoration)	
IV- Geometrical Problem of adapting overall star pattern interlace Sultan Barsbay, Khankah mausoleum 835/1432 Sultan Barsbay for Amir Ganibak	
V-First essay in Arabesque, return to zig zag Shaykh Abdallah al Manufi,, after 797/1394-95 Amir Gawhar al Qunuqbayi, before 844/1440 Amir, then Sultan, Inal 844/1451	
VI- Mastery, Refinement and luxury Amir, later Sultan Qaytbay, before 879/1474 Sultan Qansuh abu Said 904/1499 Sultan al-Adil Tumanbay 906/1591 Amir Khayrbak 908/1502	

The decoration patterns which adorn Mamluk domes play a major role in the visibility and the identification of the specific funerary complex: the first Egyptian stone domes were typically ribbed on the outside, similar to the preceding brick domes and, when the joint assemblage was not precise enough, they were plastered on the outside. The increasing accuracy in stone cutting and treatment of the joints between the stone ashlars made the practice of covering the domes with plaster obsolete. In the 1370s, the spiral ribbed decoration was developed and prevalent in many complexes. By the end of the fourteenth century, zigzag decoration was a common pattern, followed by an interlaced star pattern and floral motifs in the second half of the fifteenth century and first decade of the sixteenth; at the end of the fifteenth century and beginning of the sixteenth century, the zigzag pattern returns.

The system of decoration in Mamluk stone domes follows a pattern based on tiles, and repeats until it forms a sphere. In the layout of the decoration, the number four, following the shape of a square, and its multiples (8, 16, 32, 64) are the basis of the design **(Fig. 2)**.

In the dome of the Khayer Bek Mausoleum there is also an example of the remarkable coincidence between the ashlars and their decoration, which follows the rhythm A-B-A-B, this indicates that the complex carving pattern was created with only two different carved ashlars per ring. Several differences in the carving of details suggest that most of the work was performed before the dome was assembled, with detailed finishing and touch-up afterward. This could support the hypothesis that these domes were carved based on a drawing depicting at least one complete band that would serve as a model. This drawing, although only a bi-dimensional approximation to a single curvature, would allow the stone workers to control the carving to reproduce on each ashlar, even if in the reality the domes follow a double curvature.

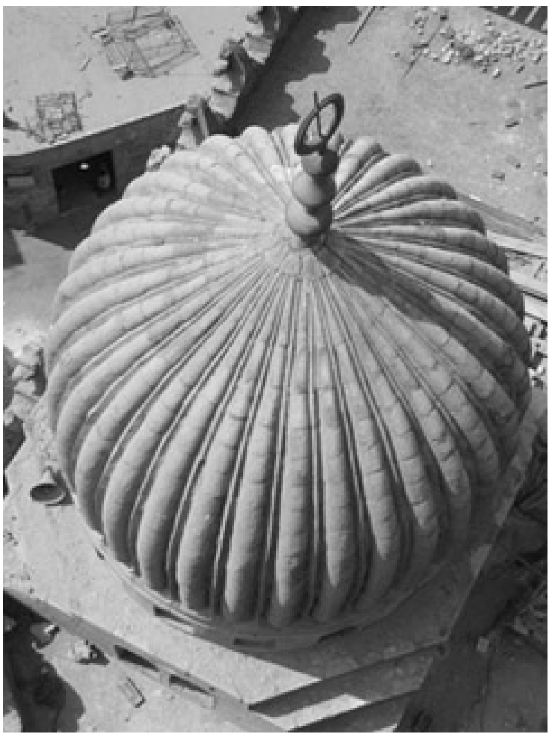


Figure 1. Mausoleum of Umm Sultan Sha'ban, aerial view. The picture clearly shows the ashlars constituting the rings decreasing in size towards the top. Photo courtesy of the Aga Khan Trust for Culture.

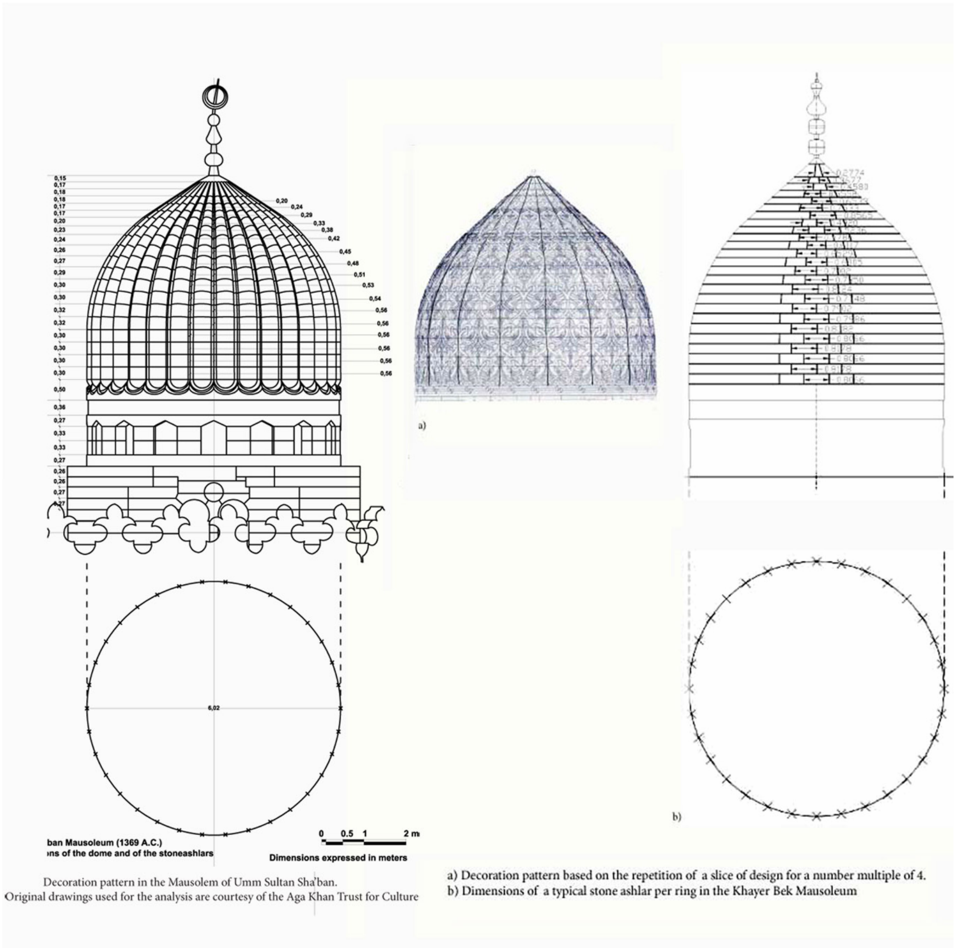


Figure 2. System of decoration in Mamluk domes

The archival research, in conjunction with the observations in the field, and the documentation produced during past and current restoration campaigns provide unique material on which to construct a sophisticated structural analysis for these three stone domes.

STRUCTURAL ANALYSIS

Given their geometry, Mamluk domes are particularly daring structures. To analyse the structural behaviour of these masonry domes, we have considered three methods: arch analysis, in which the dome is modelled as a series of radial arches; the membrane theory, through analytical equations and graphical analysis; and finally, a modified thrust line analysis by way of a new graphical and parametric computer program developed at MIT to analyse domes of conventional and non-conventional geometries.

To demonstrate one possible equilibrium condition, we assumed limit state conditions; that is, in accordance with the classical Heyman assumptions, the masonry has no tensile strength, infinite compressive strength due to low internal stresses, and sliding failure does not occur. In addition, the analyses rely on the lower bound principle, or the safe theorem: if the analysis finds a thrust line within the structural thickness of the dome section, then the structure is stable, provided that sliding failure will not occur (Heyman 1995). A masonry dome is highly indeterminate, and the analyst can only postulate possible equilibrium states. Only the structure knows the actual state of forces.

In general, two primary internal forces can occur in domes. Hoop forces in the ring direction distinguish a dome from an arch and allow construction of some domes without the use of centring. Meridional forces, which both arches and domes develop, follow the longitudes of a structure and transfer applied loads and self-weight down to the support structure.

Domex graphical analysis computer program

Using *Cabri II Plus*, a geometry-based computer software developed by Cabrilog, we developed an interactive program, *Domex*, which automates the laborious task of graphical analysis by hand (http://mit.edu/masonry). The program allows the analyst to conduct and compare in real time thrust line analyses that assume arch behaviour, membrane theory conditions, or modified thrust line behaviour.

For the three analysis methods, the program graphically outputs the location of the thrust line on the dome section in question, and calculates the corresponding horizontal thrust at the base of the dome. The program relies primarily on geometry and graphical input from the analyst to conduct the analysis on a single-screen interface **(Fig. 3)**.

The stages of an analysis with *Domex* are as follows: First, the analyst defines the section and plan geometry of the lune to be considered; a background image may be inserted into the program to assist the geometry definition. The analyst then defines the reference scale for the section, material unit weight, the reference scale of the force polygons for the graphical analysis construction, and if applicable, the allowable tensile strength of the material. The self-weight of each of the sixteen divisions in the section is determined and applied automatically at the centre of gravity of each division, with an option to apply additional surcharge at each division. The vertical axis of the force polygon, representing the gravity loads of the structure, changes in real time with changes to section geometry or loading. The program assumes loads are axi-symmetrical.

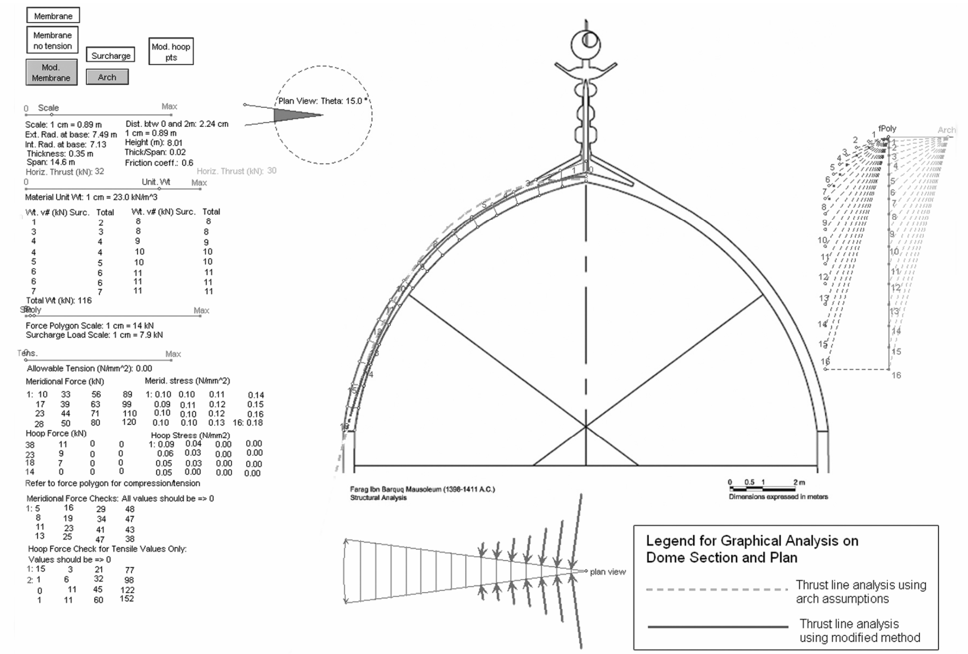


Figure 3. User interface for *Domex* comparing thrust lines based on the membrane theory and the modified thrust line conditions. Analysis shown for the dome of Farag Ibn Barquq.

Using arch assumptions, in which hoop forces do not occur, the analyst controls the unknown horizontal thrust value, permitting exploration of acceptable and unacceptable thrust line solutions. For the membrane analysis, the default location of the thrust line at the median radius provides hoop force values, shown on the dome plan view, necessary to constrain the thrust line to this location. In this method, tensile or compressive hoop forces may occur depending on the geometry and loads of the dome. In the graphical analysis, an optional no-tension solution is available in which the analyst combines a zero-hoop force thrust line with the portion of the membrane theory thrust line that incorporates compressive hoop forces; a thrust line that satisfies the no-tension condition of the limit analysis assumptions is the result.

The modified thrust line analysis provides the analyst with the greatest flexibility and control in defining hoop force values and subsequently, the shape of thrust line. For a given material tensile strength, the analyst may explore a set range of hoop force values. The flexibility in hoop force values is subjected to checks against outward sliding, which may occur when compressive hoop force resultants exceed the frictional resistance on the bottom face of each division. The analytical checks are based on the following from D'Ayala (2001):

$$\left|T_i^{mer}\right| \leq N_i^{mer} \mu$$

where T_i^{mer} represents the tangential component of the meridional force resultant at the bottom face of division i, N_i^{mer} is the normal component of the meridional force, and μ is the static coefficient of friction of the masonry. The program also includes a check against lateral sliding should tensile hoop forces be considered.

The graphical modified thrust line analysis allows the thrust line to deviate from the median radius, and accounts for the ability of hoop forces to range in magnitude; thus an infinite number of thrust lines within the structural thickness may be possible.

By interacting with the thrust line location, the analyst is able to attain a potential thrust line location that satisfies static equilibrium without the meticulous task of conventional graphical analysis by hand. It also allows more flexibility than the membrane theory equations that are difficult to apply to structures with non-spherical geometries and do not allow the forces to vary within the thickness of the structure.

General assumptions

In addition to limit state assumptions, we assumed the material weight of the stone masonry (ashlar) is 23 kN/m^3 for the three masonry dome case studies. Geometric parameters for each dome are defined below **(Fig. 4)**.

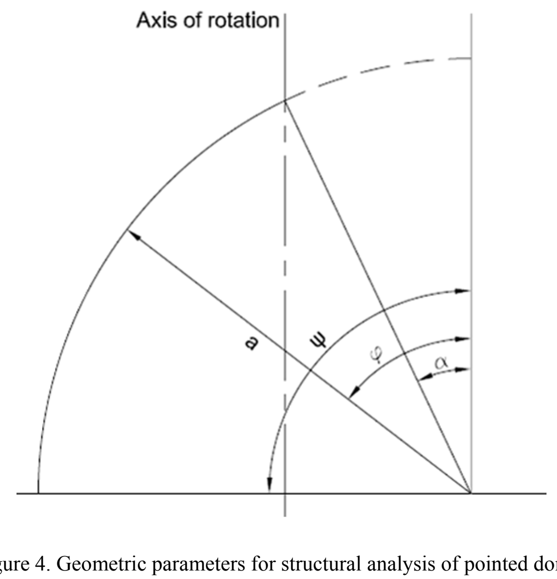


Figure 4. Geometric parameters for structural analysis of pointed domes

To simplify the analysis of these complex curved surfaces, the domes were divided into lunes, or wedge-shaped half-arches, with a value of θ, the included angle in plan, equal to or less than 15 degrees.

Arch analysis

In the arch analysis, the domes are assumed to act as a radial series of lunes pushing against their counterpart. One advantage of this analysis is that, when performed graphically with force polygons, only one unknown exists: the horizontal thrust of the arches. Using *Domex*, we obtained thrust lines for the three case studies under arch behaviour **(Fig. 5)**.

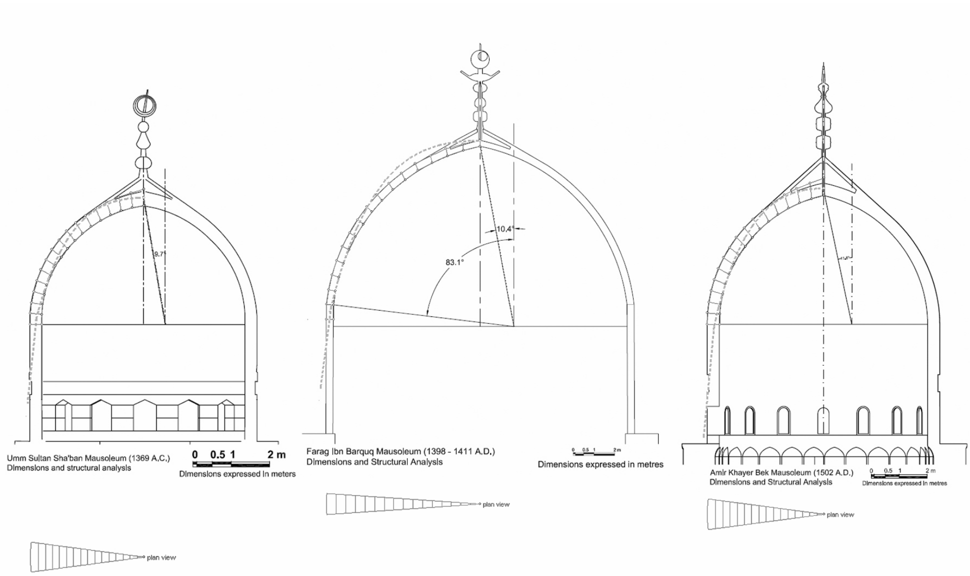


Figure 5. From *Domex*, the thrust line results under arch assumptions for the domes of, from left to right, Umm Sultan Sha'ban, Farag Ibn Barquq and Amir Khayer Bek

For the domes of Umm Sultan Sha'ban and Amir Khayer Bek, the thrust lines generated using arch assumptions stay within the dome section, but exit the cylindrical drum wall below the dome, with resulting horizontal thrust values at the bases of the domes of 3.3 kN/m (230 lbs/ft) and 8.2 kN/m (560 lbs/ft), respectively. As a result, though hoop forces are not critical for the stability of the domes, the internal forces transferred to the drum wall below will lead to an unstable structure if this masonry wall strictly adheres to limit state assumptions and if we assume an active method to resist tensile hoop forces, such as a metal tension ring, does not exist.

For the dome of Farag Ibn Barquq, the thrust line exits the dome section. Therefore, the dome of Farag Ibn Barquq will not stand without hoop forces unlike the previous domes and other well-known domes such as St. Peter's in Rome and St. Paul's in London. This distinction is likely a result of the dome's thickness-to-span ratio of 2%, significantly lower than 5% and 6% for the domes of Umm Sultan Sha'ban and Amir Khayer Bek, respectively.

Membrane theory analysis

To begin to consider the hoop forces in the domes, a membrane theory analysis using analytical equations was carried out (Billington 1982). This is the common method of dome structural analysis today. In addition, we also used existing graphical analysis methods that consider hoop forces. In both membrane theory-based methods, the thrust line is assumed at the median radius of the dome thickness, forming a membrane surface of equilibrium. The known geometry of the membrane allows for the straightforward derivation of internal force equilibrium equations. However, constraining the thrust line to the median radius equates to a dome with zero thickness; as a result, thrust line solutions potentially rely on tensile hoop forces.

The graphical method has more flexibility than the equation-based method because it permits the analyst to piecewise an alternative thrust line that meets limit state conditions to the portion of the thrust line at the median radius that does not require tensile hoop forces (Wolfe 1921). This alternative thrust line relies on zero hoop forces, and is derived in a manner similar to that of the thrust line formed using arch assumptions. Using *Domex*, we attained thrust lines for the three case studies under membrane conditions **(Fig. 6)**.

The of the three case studies analysis based on membrane theory suggests that significant tensile forces are required when φ is greater than 60 degrees **(Fig. 7)**. Graphical analysis provided slightly different force values, but indicated tensile hoop forces developing at similar φ-values **(Fig. 6)**.

The analyses based on the membrane theory show that the domes would not stand under limit state conditions. The graphical membrane theory results in unsatisfactory thrust lines that exit the domes just above the bases; in order for the domes to be stable, the thickness of their bases must increase to contain the thrust line, or be buttressed around the drum of the dome. The membrane equations calculate tensile stresses in the lower halves of the domes; although the values appear insignificant, less than 0.08 N/mm^2 (11 psi), it would be unsafe to rely on masonry to resist any tensile forces.

Modified thrust line analysis

Using the modified thrust line method, we obtained thrust lines for each dome that satisfy limit state conditions, sliding checks described above **(Eq. 1)**, and remain within the dome section **(Fig. 8)**.

After the shape of the thrust line is defined by the analyst, *Domex* calculates the corresponding meridional and hoop force values **(Fig. 9)**.

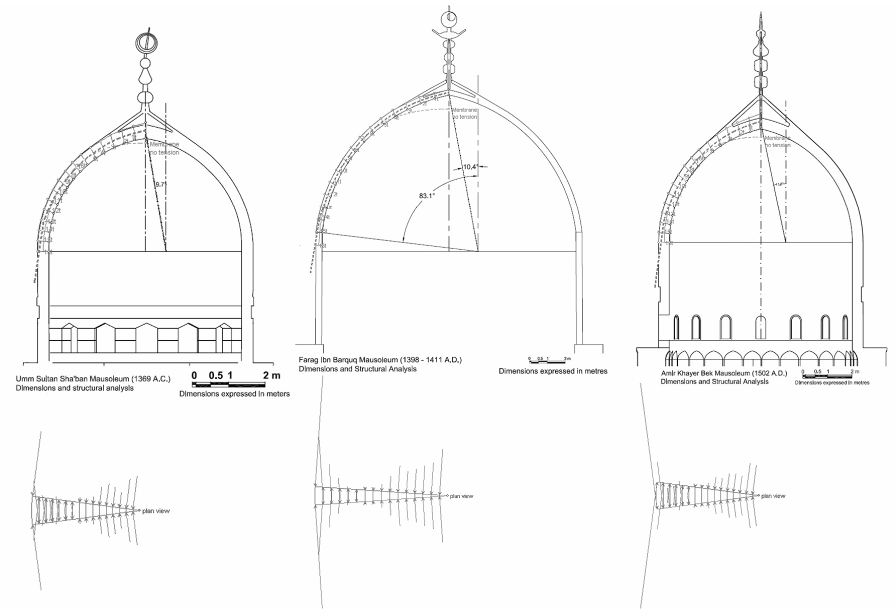


Figure 6. From *Domex*, the no-hoop force combined thrust lines under membrane conditions for the domes of, from left to right, Umm Sultan Sha'ban, Farag Ibn Barquq and Amir Khayer Bek.

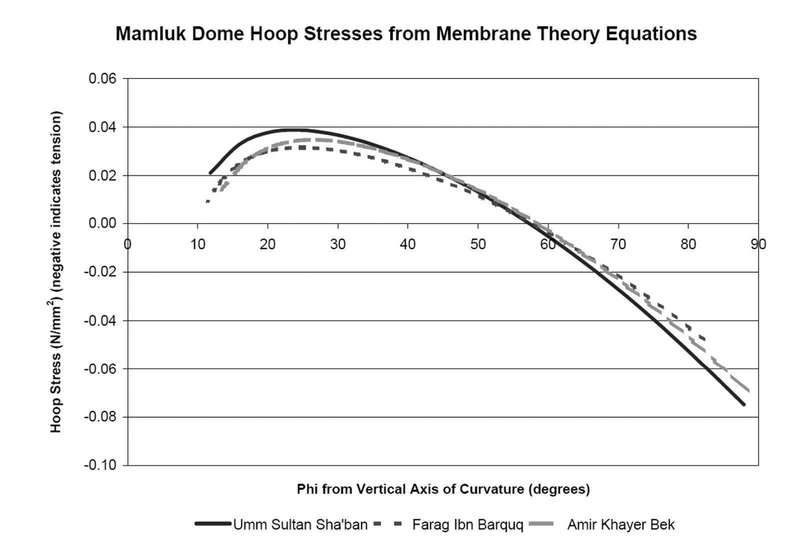


Figure 7. Hoop stress results from the membrane theory equations for the three case studies

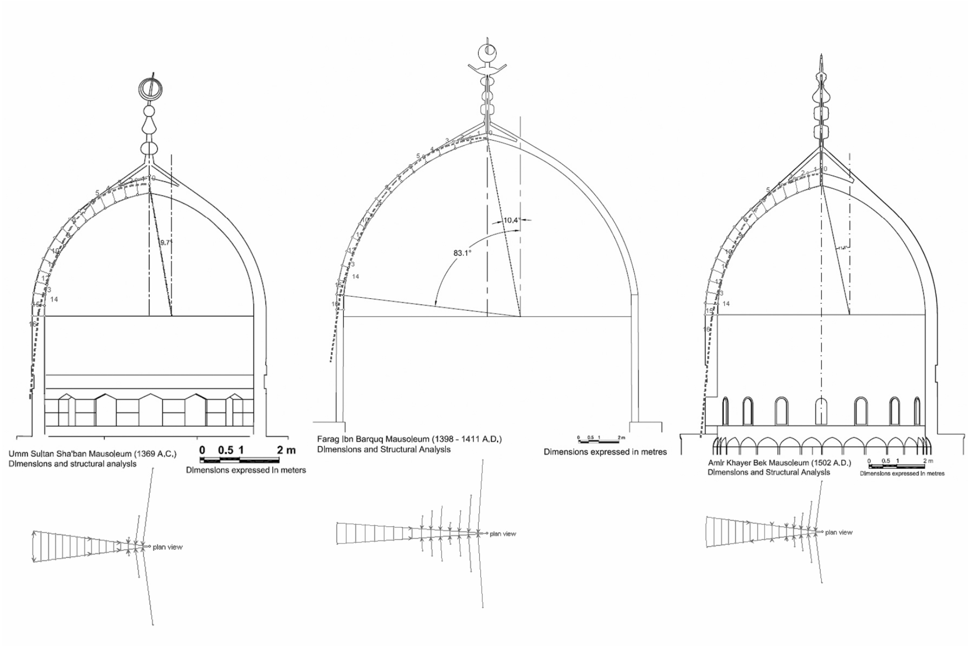


Figure 8. From *Domex*, the thrust line results using the modified thrust line method for the domes of, from left to right, Umm Sultan Sha'ban, Farag Ibn Barquq and Amir Khayer Bek

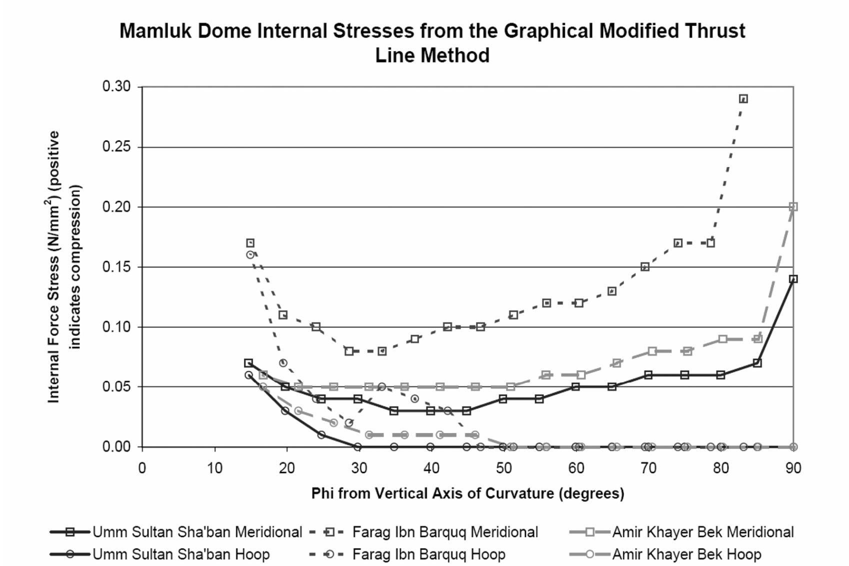


Figure 9. Meridional and hoop stress values determined for the Mamluk domes using the graphical modified thrust line method. *Domex* provides stresses rounded to the nearest 0.01 N/mm^2.

The maximum compressive stress values are 0.18 N/mm^2 (26 psi) within the domes, and 0.30 N/mm^2 (43 psi) in the cylindrical drum wall supporting the domes; both are far below the crushing stress for stone (30 N/mm^2). Thus, unlike the previous two analysis methods discussed, it is only the modified thrust line method that attains thrust lines that lie within the dome sections for all three Mamluk dome case studies. The modified thrust line method leads to thrust lines that visually demonstrate why these daring domes are standing.

However, similar to the arch analysis, the thrust lines exit the cylindrical drum support wall below the domes. At the transition from the domes to the cylindrical drum support structures, the domes exert a horizontal outward thrust that may cause the dome to splay. To keep the thrust line from exiting the structure, either the dome or the wall must resist the horizontal thrust so only a vertical reaction is transferred into support structure below. Though limit state conditions preclude masonry to be capable of resisting tension, it remains uncertain whether the domes contain an element to resist this tension force, such as a metal ring. Existing cracks, albeit minor, in the meridional direction for the domes of Umm Sultan Shaban and Amir Khayer Bek preclude any tensile resistance from developing at the base of the domes. However, we are unaware of existing cracks in the cylindrical drum wall.

Venturing cautiously into the argument that friction between the stone masonry provides limited tensile resistance, we calculated from the modified thrust line analysis **(Fig. 8)** horizontal thrust values of 4.3 kN/m (300 lb/ft), 15 kN/m (1000 lb/ft), and 8.3 kN/m (570 lb/ft) around the bases of the domes of Umm Sultan Sha'ban, Farag Ibn Barquq and Amir Khayer Bek, respectively. The normal load at the base, essentially the domes' self weights, equals 22 kN/m (1500 lb/ft), 60 kN/m (4000 lb/ft), and 46 kN/m (3000 lb/ft), respectively; a conservative coefficient of friction for the stone could provide the necessary friction to develop small tensile forces around the base.

The need for centring

The Mamluk domes of Cairo may have been constructed without the use of centring. Not only would erecting and dismantling centring consume significant time in the rapid construction schedule, the amount of required material, such as wood, was probably not locally available. Centring needs for pointed dome construction has been well discussed by Oppenheim (1989). Drawing from these concepts, we sought to verify that hoop forces generated from static equilibrium of in-progress, incomplete domes would negate the need for centring.

To simplify the analysis, we made the following assumptions: Dome construction was ring-by-ring, with succeeding course construction commencing only after lower courses or rings were complete and thus stable. The failure mode is rotation or toppling inward about the point-of-rotation at the toe of the completed masonry section **(Fig. 10)**. The horizontal component of the hoop force to resist toppling occurs at the top of the constructed section where inward radial motion is greatest and a point of contact between adjacent lunes occurs.

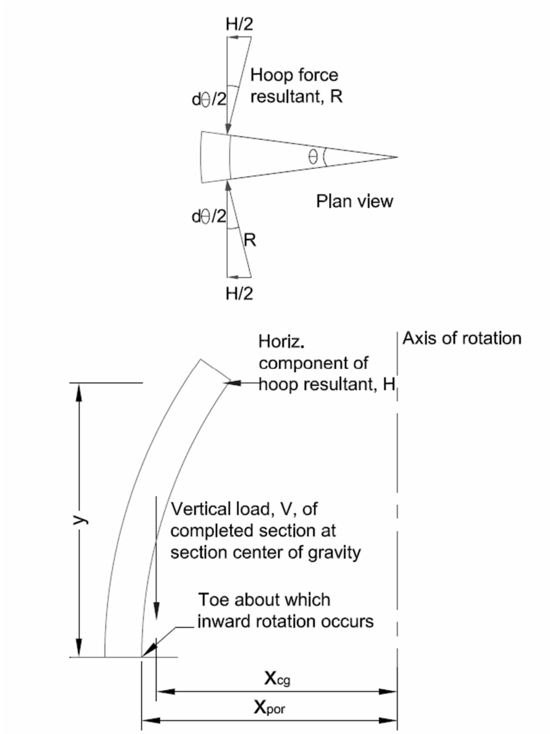


Figure 10. Parameters considered in centring analysis (after Oppenheim 1989)

The horizontal component of the hoop force, *H*, required to prevent inward rotation about the point of rotation due to the vertical load, *V*, at the section's centre of gravity was determined by a straightforward moment calculation about the toe for $x_{por} > x_{cg}$:

$$H = \frac{V(x_{por} - x_{cg})}{y}$$

The hoop force resultant, *R*, is calculated by:

$$R = \frac{H}{2\left(\sin\frac{\theta}{2}\right)}$$ where θ, the included angle in plan, is $\leq$ 15 degrees.

Each dome was evaluated at six stages of construction between φ_R, the angle at which x_{por} exceeds x_{cg}, and α. Using the modified thrust line method, we analysed the domes assuming conservatively that a single hoop force occurs at the top of the constructed dome section, similar to that shown on **Fig. 10**. With this constraint, a thrust line was fitted within the completed portion of the dome, and the hoop force resultant was compared to the minimum hoop force required to prevent inward toppling, derived from **Equations 2** and **3**. For the six in-progress stages of construction evaluated, the domes of Umm Sultan Sha'ban and Amir Khayer Bek achieved the minimum hoop force required **(Fig. 11)**.

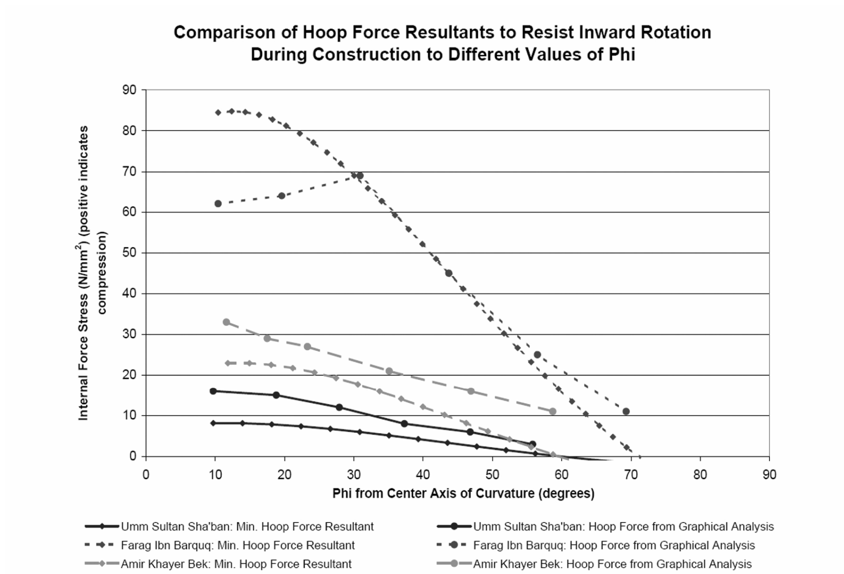


Figure 11. A comparison of the hoop force resultant required to resist inward toppling during construction to the hoop force resultant attainable through the graphical modified thrust line method

As discussed in the arch analysis, the dome of Amir Khayer Bek relies on hoop forces to attain static equilibrium. As expected, a single hoop force resultant at the top of the constructed portion does not lead to a satisfactory thrust line (i.e., static equilibrium) when $0° < \varphi < 45°$. After including more than one hoop force resultant in the upper portion of the dome to obtain a satisfactory thrust line, we found that the topmost force resultant at the temporary oculus of the dome alone does not prevent inward toppling once construction surpasses a φ-value of 30 deg. However, it can be verified that the hoop force resultants along the constructed section can collectively resist inward rotation, or:

$$\sum_{i=\psi}^{\phi_r} \left(H_i * y_i\right) \geq V_{\phi_r} \left(x_{por} - x_{cg}\right)_{\phi_r}$$

The value of y_i, the moment arm of the applied horizontal hoop force component, H, varies with the location of the thrust line within the dome thickness. We assumed H to lie at the dome's intrados. However, if H occurs elsewhere in the section thickness, for any fixed value of φ, y would increase. Therefore, the three domes studied could have been constructed without centring due to the development of hoop forces. Temporary measures to prevent inward sliding of the stone ashlars, such as mortar or rakers, would be required during construction.

However, painting on the stone ashlars in the interior of Farag Ibn Barquq dome and carvings on the topmost course of Amir Khayer Bek and Umm Sultan Sha'ban suggest the use of some method of scaffolding. The collapsed remains of other domes have a layer of brick below the stone ashlars, indicating that in this case a centring for the domes was provided by an inner brick dome **(Fig. 12e)**. In other cases minor centring could have been used to complete the assemblage of each ring **(Fig. 12d,f)**.

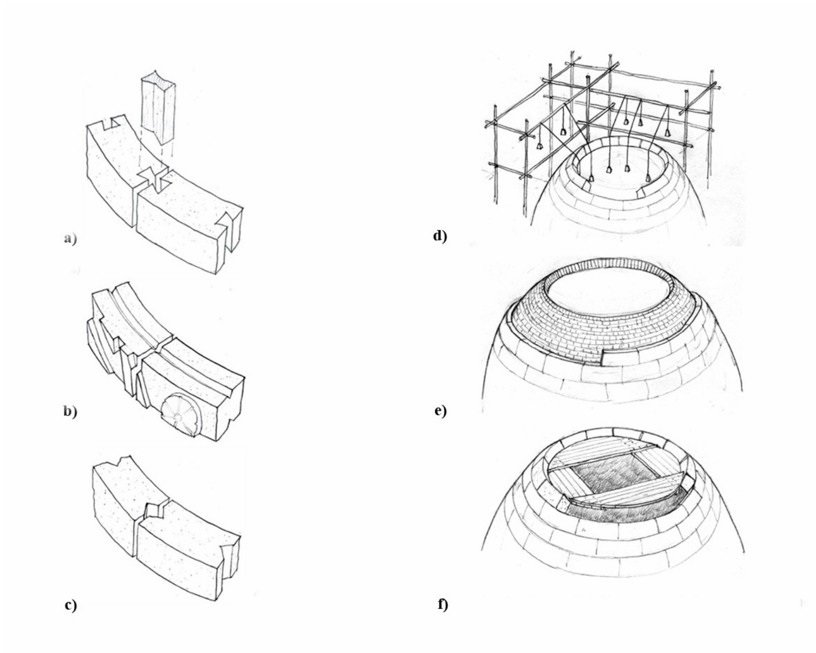


Figure 12. Connection examples between stone ashlars to resist the tensile hoop force. Reconstruction based on restoration reports (a, b) and fieldwork (c). Three possible reconstruction of the dome assemblage (d, e, f)

FINDINGS FROM THE RESTORATION FIELDWORK

We have used old restoration reports and current results from restoration fieldwork carried out by the Aga Khan Trust for Culture (www.akdn.org/agency/aktc_hcsp.html) on two Mamluk domes to augment our structural analysis with accurate drawings, and to verify some dome features that can only be seen either when they are dismantled or when scaffolding is used.

The findings reveal several common structural features for Mamluk stone domes:

- The double profile of the inner and outer domes at the crown constitutes a discontinuous double dome while the space between the two profiles is filled with earth and rubble. The inner geometrical form differentiates from its outer form, as indicated by the section and by the three dimensional reconstruction **(Fig. 13)**.
- The conic termination of the domes hides a hollow metal spire holding the crescent moon characteristic of all Mausolea and mosques.
- The absence in the early stone domes of horizontal connections between the ashlars constituting the annular rings and their introduction in fourteenth century stone domes. In the Mausoleum of Farag Ibn Barquq, a late 1960s report of the restoration work on the dome attests the presence of connectors shaped as dovetails **(Fig. 12a)**. These teak connections are embedded in the voussoir stones; the report does not mention if they are present only in one region of the dome.
- Calcium sulphate ($CaSO_4$) mortar, which has a quick set time, in the construction process. (microscopic and chemical analysis performed in the MIT department of material science on the mortar samples from the Amir Khayer Bek dome upper course). Stone blocks limestone, Calcium Carbonate, ($CaCO_3$) populated of 2mm fossils in Khayer Bek upper course. This findings excluded the possibility that the identical stone ashlars were cast.

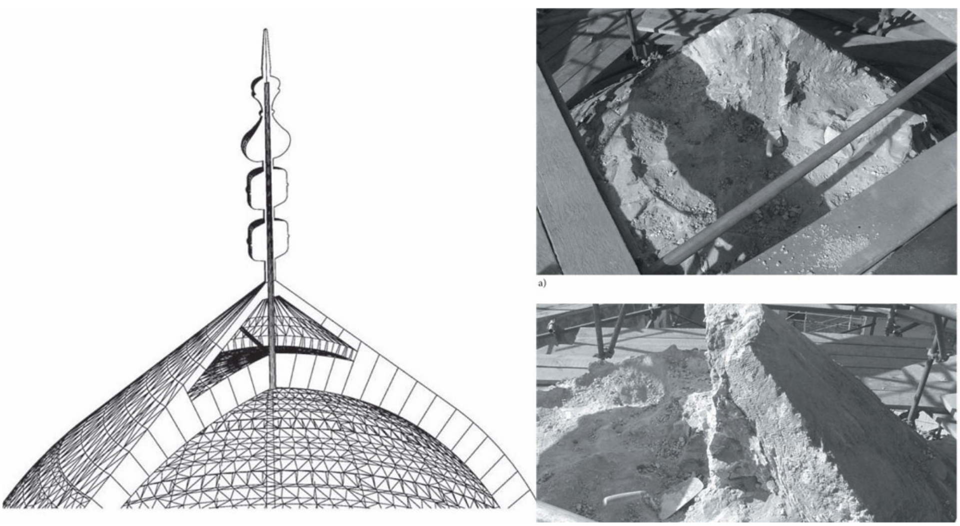


Figure 13. Three-dimensional reconstruction of the dome of Khayer Bek based on the restoration fieldwork

From our site surveys, we noted differences in the patterns in the dome cracks between the domes of Umm Sultan Sha'ban, where step cracks reaching 3 to 4 cm wide occur, and Amir Khayer Bek, where minor longitudinal cracks occur. In the former, external cracks and internal cracks align; in the latter, they do not correspond. The change in cracks suggests techniques in cutting and assembling the stone ashlars improved over time. This may also suggest the presence of horizontal and vertical connections between ashlars in Amir Khayer Bek; this will become known when the restoration work is completed.

CONCLUSIONS

The construction of Mamluk domes in Cairo in the fourteenth and fifteenth centuries was the product of several factors, including the patron's desire to protect his tomb with solid and ornate roofing that would remain visible from afar. Moreover, the uniqueness of the decoration pattern guaranteed a unique association between each Mausoleum and the Sultan or Amir who is buried there, a tradition that continues today in Cairo. The shape and the decoration of the domes served these purposes, while the structurally unnecessary conical crown that progressively elongated the domes validates an aesthetic aim behind the construction features of these complexes.

The three dome case studies share several features in their stereotomy and decoration methods, indicating the structural understanding and aesthetic principles of domes were widespread. However we have been extremely fortunate in that ongoing restoration work provided other details. A better understanding of these numerous but overlooked structures can only come from the detailed study of their individual technology and a comparative structural analysis.

In terms of structural behaviour, the geometries of the dome generally become more daring over time in that the drum elongates, and the thickness-to-span ratios decrease. By neglecting the development of hoop forces in domes, the arch analysis method does not attain a satisfactory equilibrium solution for the dome of Farag Ibn Barquq: it would not stand. Constraining the thrust line to the median radius, as in the membrane theory, mandates the domes to rely on tensile hoop forces to develop near their bases in order to stand, a fact that cannot be readily assumed. Therefore, existing analysis methods could not explain the safe performance of these domes for over 500 years.

To investigate this problem, researchers at MIT have developed a new analysis program called *Domex*. This program, which modifies traditional thrust line analysis, attains thrust lines that satisfy the analysis assumptions and criteria for all three domes with the assumption that friction in the masonry of the drum support wall will constrain the thrust line. The modified thrust line method approaches dome analysis with the innovation of allowing internal hoop force resultants to assume a range of values that subsequently determine the magnitude of meridional forces; current dome analyses typically assume the converse.

Domex, the computer program discussed in this paper, takes dome graphical analysis to a new level of accessibility by automating the traditionally laborious procedure, allowing non-conventional dome geometries, and outputting in real time thrust lines and force values that when combined, provide a visual representation of structural behaviour that may otherwise be obscure.

Through *Domex* and the modified thrust line analysis, the paper also demonstrates that the Mamluk dome case studies could be constructed without centring because of the resistance to toppling provided by internal hoop forces that developing as the domes are constructed ring by ring. However, site surveys have evidence that some method of scaffolding was likely used in constructing these domes.

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