The Evolution of the Work of Eduardo Torroja: Shell Roofs with and without Reinforcement Rings

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INTRODUCTION

At the beginning of the twentieth century, the construction of thin reinforced shell concrete roofs was widespread in Europe. This roof is of the type where a cylindrical shell with a span of between 3.00 and 5.00 m span is built among arches that give the shape of the roof. These arches have a tie beam to resist thrusts and there is therefore only a vertical reaction on the piers. Arches are placed at the bottom side of the shell. At this period concrete was considered to be an elastic and lineal material that obeyed Hooke's law and the arches were therefore analysed in these terms.

In 1913 Eugène Freyssinet proposed a system for constructing these kinds of roofs, which involved putting the reinforcement arches in the upper side of the shell.

> This subject, which does not appear on the surface to be very important, has had a great influence in the development of this kind of construction; it considerably simplified their construction and reduced the price. By putting the reinforcement beams on the upper side, the formwork was transformed into a continuous surface, which was made easily and rapidly by workers and could be removed quickly without breaking up the cast. The reinforced beams are constructed after the vault has been constructed, by means of simple formwork.
>
> (Freyssinet 1926, p. 266)

He proposed several projects for hangars in 1913, but these were not built because of the outbreak of war. In 1916 a series of eight hangars, formed by a thin vault concrete shell, and reinforced by stiffening beams on the upper side of the vault, were constructed. In the next few years, many hangars were built in this way **(fig.1)** although some of the shells were made with different shapes, using conic shells rather than cylindrical vaults **(fig.2)**. The ratio between the span and the thickness was close to 100, and in the description of this work no references were made to problems of buckling.

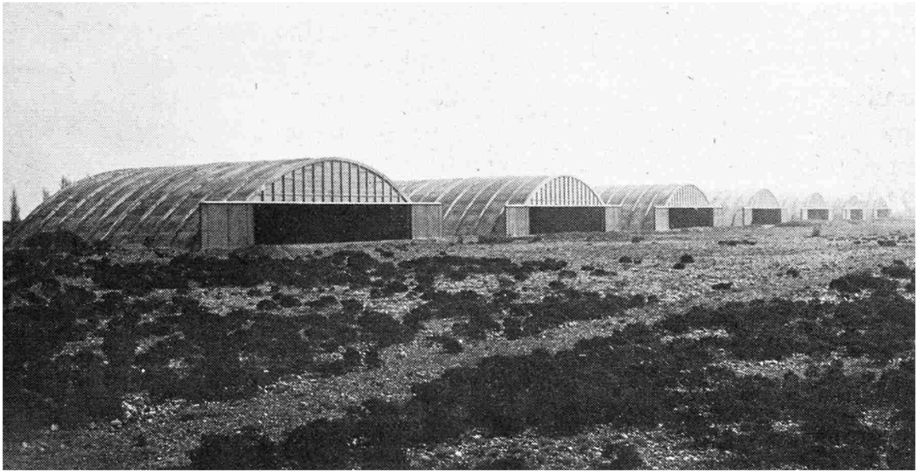

Figure 1. Hangars at Istres, France, designed by Freyssinet in 1917. The structure was a Cylindrical Vault reinforced by stiffening beams in the upper side (Fernández 1978, fig.46)

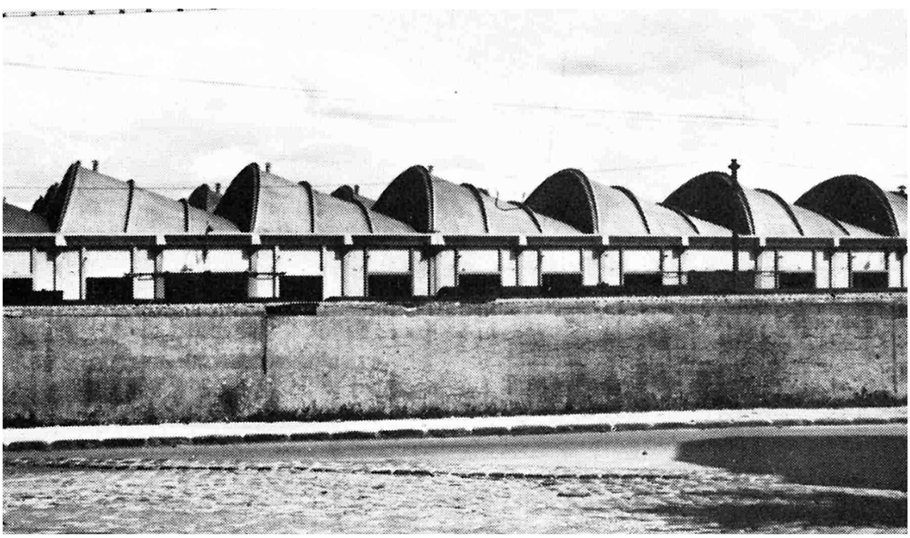

Figure 2. Railway workshop in Bagneaux, France, designed by Freyssinet. Conoidical Vault reinforced by stiffening beams in the upper side (Fernández 1978, fig.72)

Some years later, the engineer Fauconnier (1934) published his experiences of testing a thin conoidical vault of reinforced concrete with stiffening beams **(fig.3)**. One of the conclusions of this test was the description of the way in which the load is carried. Fauconnier said that the vault of 17.50 metres span, in between the stiffening beams separated 4.00 metres, supports almost 6/7 of the load working as a vault and the 1/7 remaining was supported by the stiffening beams. At the conclusion of the test, no mention was made of problems of buckling. The vault broke because of separation between the vault and the stiffening beam with a load more than ten times the calculated load.

We can see that reference to of the instability of the concrete vaults was made in the early years of this form of construction. These vaults were constructed with a thickness close to 100.

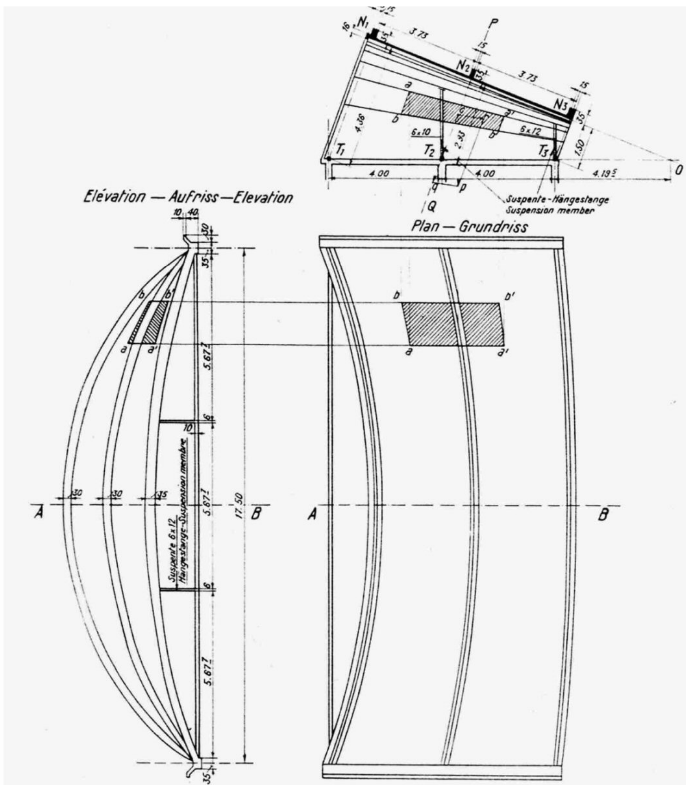

Figure 3. Design of the roof 17.00 m span, built by Entreprises Limousin and tested by Fauconnier for the Cie Métropolitain de Paris (Fauconnier 1934, fig. 2).

THE FIRST SLENDER VAULTS

In Germany, Walter Bauersfeld and Mergler, engineers at Dyckerhoff and Widmann, built the first spherical dome in concrete in 1922 for tests carried out in Jena **(fig.4)** before the construction of the Planetarium of the Deutsches Museum in Munich. In order to build the dome, they proposed installing a spherical net of steel bars and Mergler suggested projecting concrete against formwork. The spherical shape of the dome allowed the use of the same pieces of formwork again and again. The dome was analysed like a continuous surface.

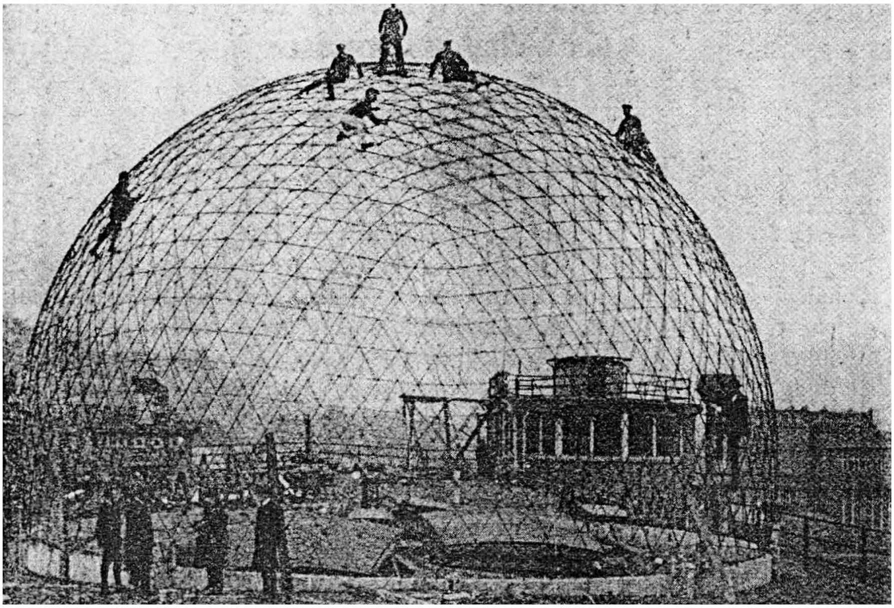

Figure 4. Dome 16.00 m span and 3 cm thick, designed by Bauersfel and Dischinger built in Jena in 1922 (Specht, 1987).

The construction of the dome at Jena was made possible by Prof. Spangenberg's report. Construction began in the winter of 1923-1924. A star shape was used for the edge, placing the bars in cantilever. The bars close to the edge started to buckle and the ends and some stabilisation bars were needed.

In this construction, Bauersfeld analysed the bending moment and deformation. In the first dome (Jena 16.00 m span), not only were the tension and compression in the plan of the dome taken into account, but bending moments and deformation were studied.

The theory of the rigid rotation dome was published by Föppl, Drang and Zwang. Second order differential equations were needed. Bauersfeld found an approach which reached a solution, in which in order to analyse the problem of buckling the Zoelly formula was used, which gives a safety factor of 13.

Bauersfeld asked Doctor Geckeler to undertake tests. He did many tests and found that loads a quarter of the Zoelly formula starts some buckling.

FIRST TORROJA'S SHELLS PROJECTS

In the autumn of 1933 Torroja begun several projects with shell structures. The first project he undertook was the roof of Algeciras Market. This was a dome 46.22 m span, supported by 8 piers. The shell consisted of a spherical concrete shell, 44.10 m radius, 9 cm thick and reinforced by eight pieces of cylindrical shell between the piers. The shell was built using wooden formwork on a scaffold. With this method there was no problem with bars buckling as had happened to Bauersfeld with the construction of his first dome in Jena.

In 1934 Flügge proposed a value for the critical load for spherical shells:

$$p_{cri} = \frac{2Eh^2}{\sqrt{3\left(1-\nu^2\right)}\cdot a^2} \qquad (1)$$

This is the value of the least uniform pressure on a spherical shell that causes the buckling of the shell. Considering a concrete with a modulus of elasticity for the concrete of $E=20\ 000\ MPa$, the radius of the shell $a=44.20\ m$ and $h=0.10\ m$, the p_{cri}=0.134 *MPa* estimate is much higher than the action on the shell including its own weight $p=0.004\ MPa$. So, we can see that the safety rate is close to 30 and, in this case, the risk of buckling is small. However the expression (1) was given for a full sphere, Bruch (1975, 214) used the same for spherical caps such as the Algeciras market roof.

In 1934 Torroja started to design cylindrical shells. These were the first shells structures built in Spain. The first cylindrical shell was a roof for a gymnasium in Madrid: The roof in "La Escuela Elemental de Trabajo" was built to cover a space of 22.00 m long and 8.00 m wide in the court of a existing building. The type of structure commonly used in such a situation was formed by a series of beams and reinforced concrete slabs on many piers. This represented about 500 kg/m^2 in weight. Additionally, in this case excavations of up of five metres deep were needed for the foundations of the piers (Torroja 1934).

Torroja proposed a concrete shell structure: a cylindrical shell whose guideline was an ellipse. This structure, a 5 cm thick shell was only 125 kg/m^2 in weight, four times less than the commonly used structure. This light structure could be suspended from the façade of the existing building by four cables **(fig.5)**. This eliminated the need for the excavation of any foundations. In this particular case, the use of a shell structure was decided primarily for financial reasons. The new structure needed a quarter of the amount of material that was normally used for beams and piers. The surface of formwork was less in the new structure and, there was no need to build piers or excavate foundations.

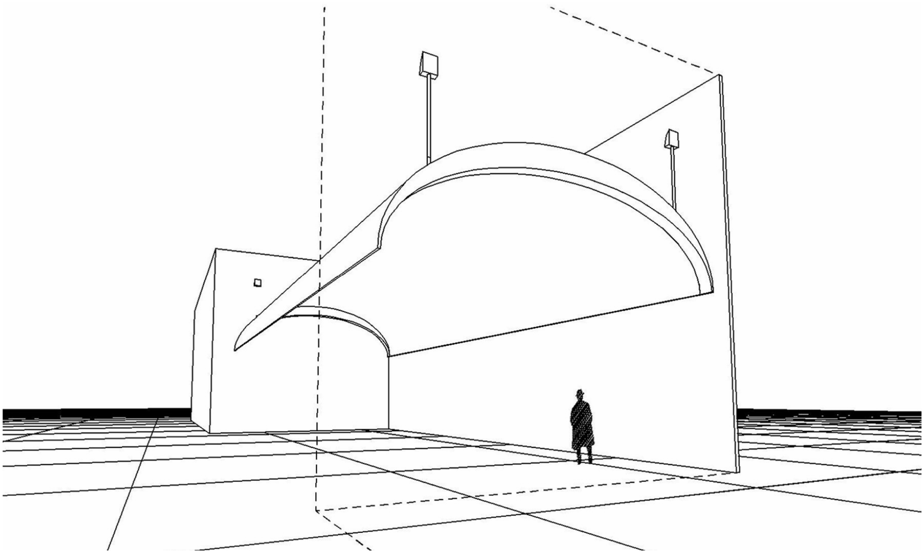

Figure 5. View of the roof of the Escuela Elemental de Trabajo. The cables from which the shell was suspended can be seen.

The shell designed by Torroja was 22.00 m in height, 8.00 m wide and 8 cm thick; the guideline was an ellipse. For the analysis of the shell, the expressions given by Dischinger (1928) were used. In this text, the question of buckling was not treated and Torroja did not study this problem in his first projects. For the analysis Torroja only considered the membrane forces and the expressions used were the well known:

$$N_{\varphi} = -rZ \tag{2}$$

$$N_{x\varphi} = -\int \left(Y + \frac{1}{r} \cdot \frac{\delta N_{\varphi}}{\delta \varphi} \right) dx + f_1(\varphi) \tag{3}$$

$$N_{x} = -\int \left(X + \frac{1}{r} \cdot \frac{\delta N_{x\varphi}}{\delta \varphi} \right) dx + f_2(\varphi) \tag{4}$$

expressing the radius of the ellipse as:

$$r = \frac{a^2 b^2}{\left(a^2 \cdot \sin^2 \varphi + b^2 \cdot \cos^2 \varphi\right)^{3/2}} \tag{5}$$

and the weight of the shell 5 cm thick 125 kg/m^2, and for snow 60 kg/m^2 with a law like *60cos*$^2\varphi$ then the loads on the shell are:

$$Y = 125 \cdot \sin\varphi + 60 \cdot \cos^2 \varphi \cdot \sin\varphi \tag{6}$$

$$Z = 125 \cdot \cos\varphi + 60 \cdot \cos^3 \varphi \tag{7}$$

The maximum compression N_x is 25.50 kg/cm^2, in the crow.

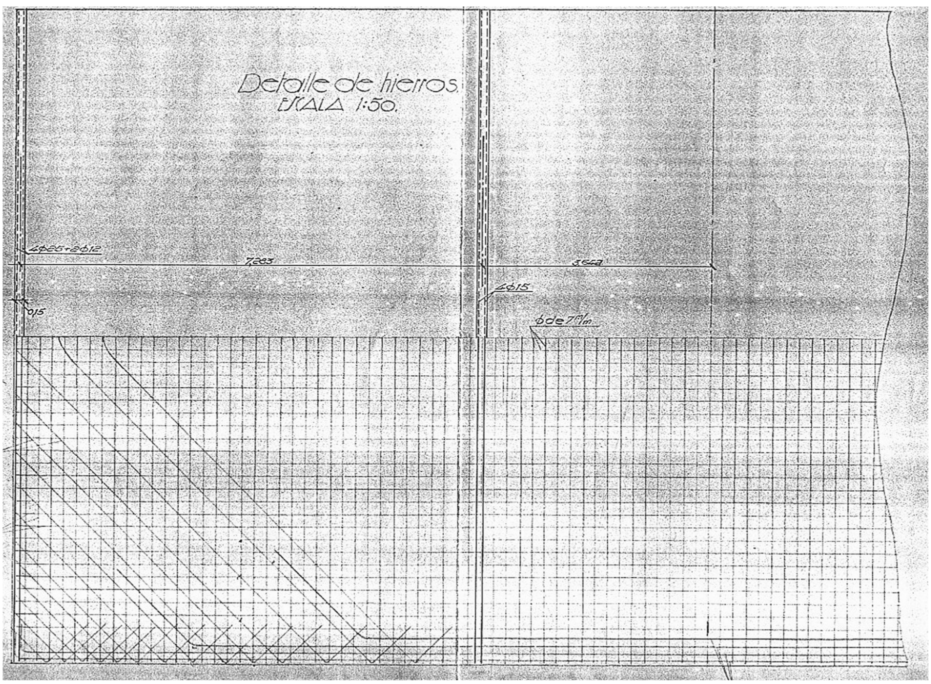

Figure 6. Plan of the roof. It shows the beam at the end of the shell and the stiffening beam placed at a third of the span (AET, file number 275)

At that time many vaults were built by Diwydag, the firm where Dischinger worked. In many of them the span was higher than the 22.00m of Torroja's project and the question of buckling was not established. Despite this, Torroja planned to construct two stiffening rings in the upper side of the surface as can be seen in the plans of the project. These stiffening rings are two beams 50 cm high and 20 cm wide as shown in **(fig.6)**. When the structure was finished, a test was planned. The test is shown in **(fig.7)** and we can see the steel bars prepared to construct the stiffening ring. We do not have information about the results of the test, but the results must have been satisfactory because as is shown in **(fig.8),** a photograph taken after the test was finished, the stiffening rings were not built. From that we can assume that Torroja believed that there was no danger of instability in cylindrical shells with these proportions.

At this time Flügge (1934) also proposed a value for critical pressure for cylindrical shells:

$$p_{cr} = k\frac{Eh}{a}10^{-6} \tag{8}$$

as is shown in Bruch (1975, 165) where k is a factor depending on the ratio between the span and the radius of the shell (L/a), and the ratio between the radius of the shell and its thickness (a/h). In **(table 1)** different values of p_{cr} obtained using this expression for different projects designed by Torroja in 1935.

Figure 7. View of the test carried out during the construction of the shell. We can see the steel bars prepared for building the stiffening beam (AET, file number 275).

Figure 8. View of the shell built without the stiffening beam after the load test was finished with satisfactory results (AET, file number 275).

THE VAULT OF THE FRONTÓN RECOLETOS

After the first test of a cylindrical vault made in the first half of the year 1935, Torroja collaborated with the architect Secundino Zuazo in the design of the roof of the "Frontón Recoletos" **(fig.9)**. The plan of this building was a rectangle 55.00 m long and 32.50 m wide. A cylindrical shell was designed for the roof. This shell was composed of two cylinders with different radii, the larger 12.20 m and the smaller 6.40 m. The two cylinders were joined as shown in **(fig.10)**. The shell was 9 cm thick. On the north side of the roof, the continuous shell ended and was replaced by a barrel vault. This barrel vault was formed by beams 1.50 m length and 0.30 m x 0.15 m section. The roof was supported by the perimeter walls.

The analysis of the structure was made without taking into account the barrel vault, and the shell was considered as continuous. The maximum tension in compression was close to 50 kg/cm^2 near to the crown of the cylinder. The maximum pressure on the roof, considering its own weight, the wind and the snow was 415 kg/m^2. For the wind and the snow a variable law was considered.

A description of the project was published by Torroja in1942, but no mention was made of the study of buckling.

The roof was finished in January 1936, but by August 1939 when the Civil War ended, the shell was seriously damaged. During the war many bombs exploded near the shell, some holes were made **(fig.11)**, and important displacements happened.

Figure 9. View of Frontón Recoletos. Architect: Secundino Zuazo, Engineer: Eduardo Torroja (AET, file number 277).

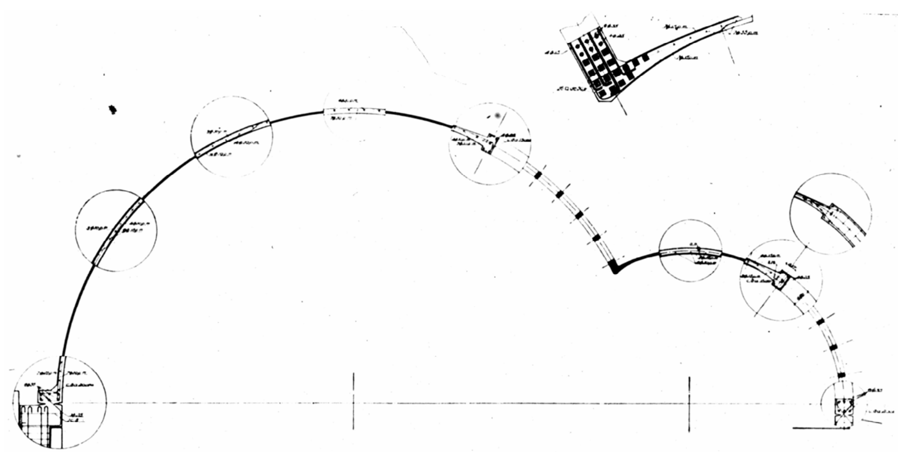

Figure 10. Transverse section of the roof of Frontón Recoletos. We can see how the problem of support in the boundaries was solved and how the edge between the two lobes was reinforced (AET, file number 277).

Figure 11. View of a hole made by a bomb during the Spanish Civil War.
(AET, file numbers 277).

To rectify this situation, Torroja proposed a reinforcement of the shell consisting of constructing some rings on the big lobe **(fig.12)** provided with turnbuckles. Those rings would allow the original shape of the shell to be reroofed and, on the other hand, increase the transversal stiffness of the big lobe, since this had caused the failure of the shell: The big lobe had had insufficient stiffening and suffered two longitudinal cracks, becoming deformed and causing the failure.

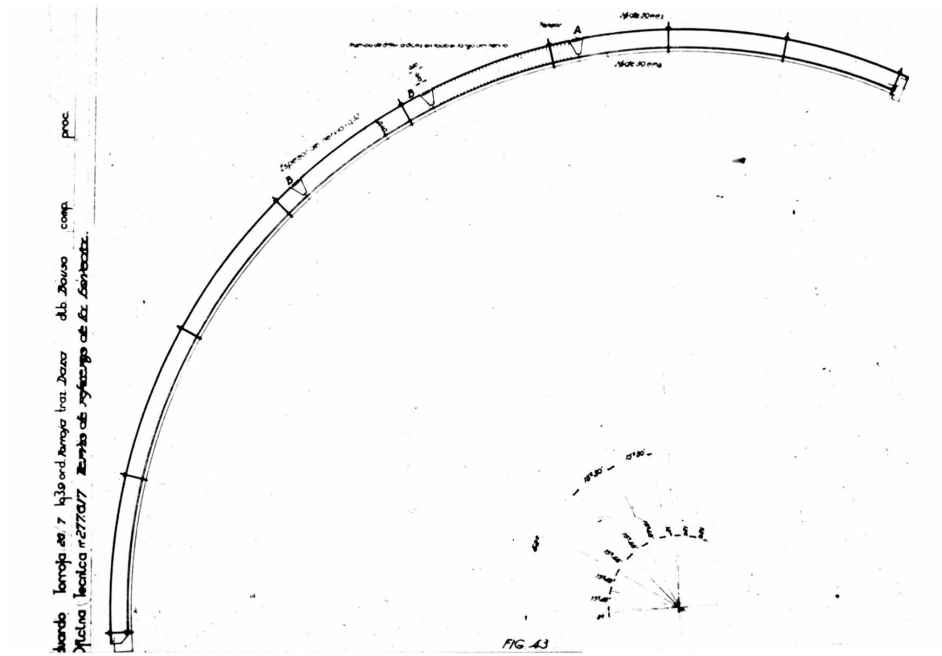

Figure 12. Plan of the reinforcement proposed by Torroja in July 1939 after the end of the Civil War. The reinforcement consisted of a series of stiffening beams (AET, file number 277).

Föppl (1930) gave an expression for calculating the risk of buckling of a cylindrical surface. He gave the required thickness as a function of the pressure that the surface needed to support and the radius of the cylinder:

$$p_k = \frac{E}{4}\left(\frac{h}{r}\right)^3$$

where E is the modulus of elasticity, h is the thickness of the shell, and r the radius of the section. Applying this expression, we have a safety factor of 5 for the vault of Areneros and the vault of the Fronton Recoletos had a safety factor of 0.88.

CYLINDRICAL VAULT PROJECTS AFTER 1940

The San Paulo Gimnasium, used a 105.00 m span dome for the roof. This project was never built but in the previous calculus, Torroja studied some possible solutions. One consists of a spherical shell 19.00 m high. For the analysis the possibility of buckling was calculated using the expression given by Flügge:

$$q_{\min} = 2\sqrt{\left(1-\nu^2\right)R} - 6\nu R$$

where

$$R = \frac{K}{Da^2}$$

$$D = \frac{ES_1}{1-\nu^2}$$

$$K = \frac{EI_1}{1-\nu^2}$$

and

$$R = \frac{EI_1}{\frac{ES_1}{1-\nu^2} a^2 \left(1-\nu^2\right)} = \frac{I_1}{S_1 a^2}$$

$$p_{\min} = \frac{2Dq}{a} = \frac{2\cdot\left(2\sqrt{\left(1-\nu^2\right)\frac{I_1}{S_1 a^2}} - 6\nu\frac{I_1}{S_1 a^2}\right)\cdot\frac{ES_1}{1-\nu^2}}{a} = \frac{4\sqrt{\left(1-\nu^2\right)\frac{I_1}{S_1}}\left(\frac{ES_1}{1-\nu^2}\right)}{a^2} - \frac{12\nu I_1 ES_1}{\left(1-\nu^2\right)\cdot S_1 a^3}$$

and

$$p_{\min} = \frac{4\sqrt{\frac{I_1}{S_1}}ES_1}{\sqrt{\left(1-\nu^2\right)}a^2} - \frac{12E\nu\cdot I_1}{\left(1-\nu^2\right)a^3}$$

Some values of the critical load as proposed by different authors:

Author	Year	$q_{cri} = \gamma E \dfrac{S^2}{R^2}$
Zoelly	1915	$\dfrac{2}{\sqrt{3\left(1-\nu^2\right)}} = 1.156$
V. Kármaán y Tsien	1939	0.3652
Friedrichs	1941	0.9
Tsien	1942	0.312
Muschtari y Surkin	1950	0.34
Feodosjew	1954	0.32
Schmidt	1960	0.2
Haas y Van Koten	1966	0.34
Del Pozo	1973	0.3168

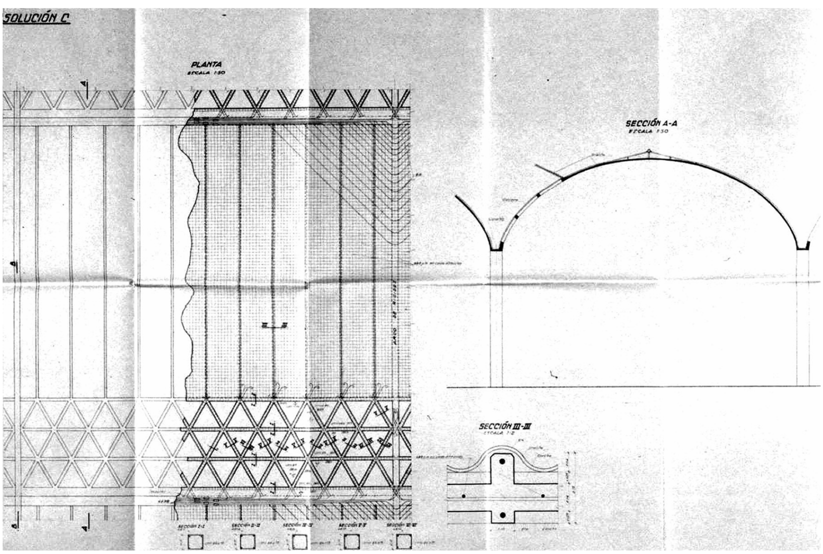

Figure 13. Proposal for an industrial building. The proposed roof was a cylindrical vault whose guideline was a cicloide. The shell had two metre stiffening beams (AET, file number 665).

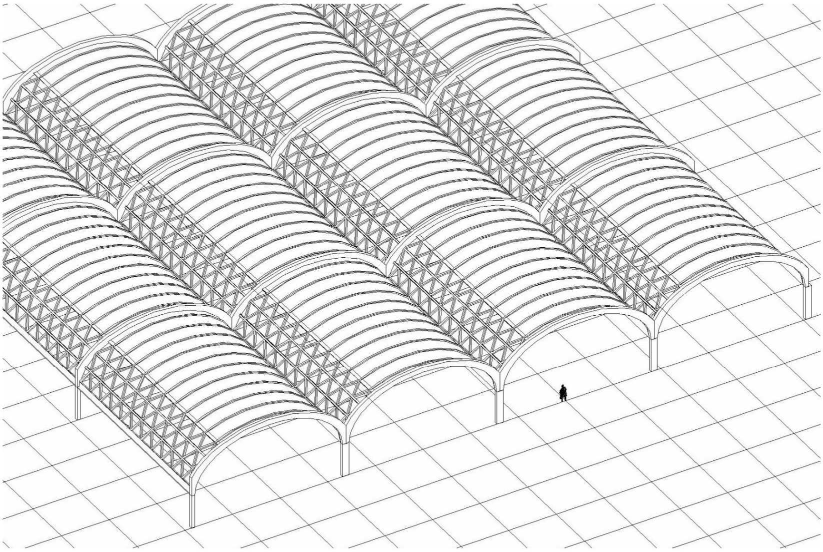

Figure 14. View of a proposed structure for an industrial building.

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Consultar Kurt Beyer. La estática en el hormigón armado.

Expresión del empuje de un arco:

$$H = \frac{pl^2}{8f(1+\nu)}$$

en donde $\nu = \frac{15}{8}\frac{1}{f^2}\frac{I}{A}$ y $M = \frac{pl^2}{8}\frac{\nu}{1+\nu}$

empleadas en el anteproyecto del palacio de los deportes, exp. Nº 761

f es la flecha del arco, *I* es la inercia, *A* el área.