Analysis of the Sub-Rafter *(Sous-Arbalétrier)* in French Medieval Timberwork

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SCOPE AND OBJECTIVES

The present study is focused on a particular roofing timberwork element, named *sous-arbalétrier* in French, and that we will call *sub-rafter* in English, defined here as an element parallel to the rafter, which it "doubles", being placed underneath its lower segment, usually between the tie-piece and the collar beam **(Fig.1)**.

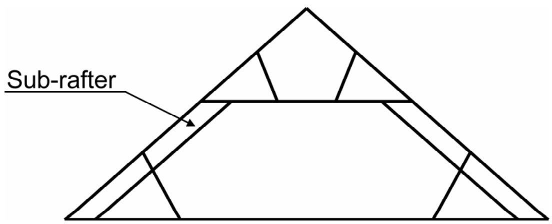


Figure 1. Sub-rafter

The oldest French timberwork specimen still standing including such a sub-rafter we know is that at Noirlac Abbaye (1150-1160). This element is seen to be used approximately up to the middle of the thirteenth century. Six examples of roofing timberworks with sub-rafters of this period are shown in **(Fig.2)**. All these are of the rafter-truss type without purlins.

Other examples could be included that show elements that could be seen as variants of the sub-rafter, either parallel to the rafter and extending all the way up, either not parallel to the rafter, and generally extending to queen posts. These are not within the scope of this study.

Existing classifications (Hoffsummer 2002) usually do not distinguish the morphological properties arising from the relative positions of the different carpentry elements, from their mechanical role implicitly attributed, nor from their denomination according to a skilful terminology. Different functions are attributed to the sub-rafter by various authors, depending on the configurations of the other elements of the stiffening system. This could be one reason for which existing classifications do not identify the sub-rafter as a discriminating element for defining morphological categories.

It can be read in LePort (1988) that this sub-rafter could serve, depending on the case, either to restrain the outward displacements of the main rafter bases and relieve its bending as in the case of

Noirlac (1150-1160), or to serve as an internal arch system in combination with the diagonal bracing as in the timberwork of St John's hospital chapel in Angers (1185), or merely to help in the erection process such as the case of Hermonville (1180) **(Fig.2)**. But the commonly accepted idea is that the sub-rafter is there to reinforce the main rafter in its lower part (Froidevaux 1986, Hoffsummer 2002).

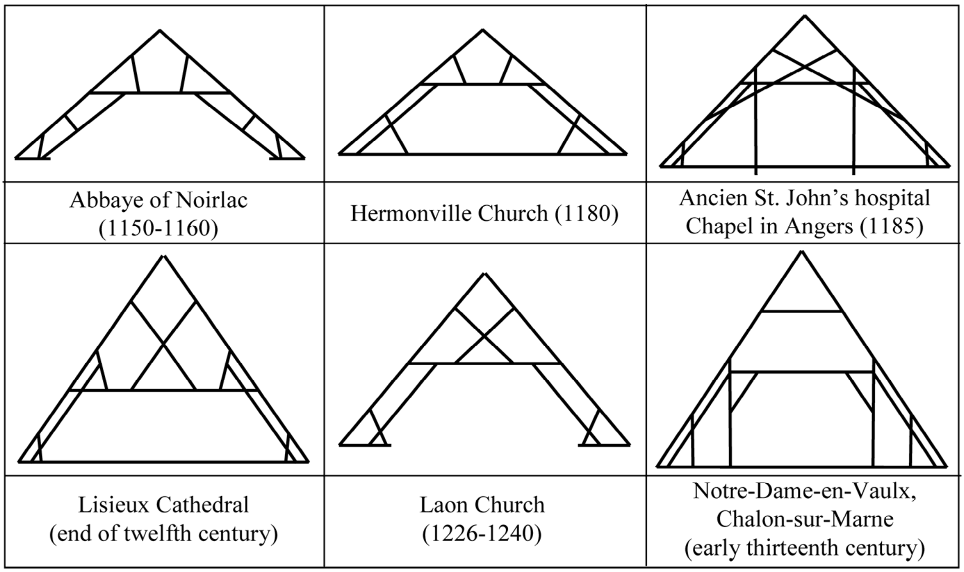


Figure 2. Examples of 12th and 13th century French timberworks with sub-rafters (not scaled)

In any case, introducing this new element increases the statical redundancy of the truss, and the flow mechanism of the forces becomes more subtle and uncertain. One needs to consider the distribution of stiffness between the different elements in order to ascertain the distribution of the internal forces. A rigorous model for the mechanical behaviour of the structure is needed to reach reliable conclusions.

The object of this study is to identify which characteristics of the geometric pattern of the truss appear to be relevant with respect to the mechanical function of the sub-rafter. The conclusions are aimed to throw some light on the research process of the designer-builders of that time.

METHODOLOGY

The methodology is based on the analysis of a reference configuration, in which variants are introduced to cover the different strategies employed in the rafter footing system **(Fig.3)**. On the one hand, this issue is a fundamental one for equilibrium, and on the other hand, the different treatments of the base lead to different generic behaviours of the corresponding basic rafter-trusses.

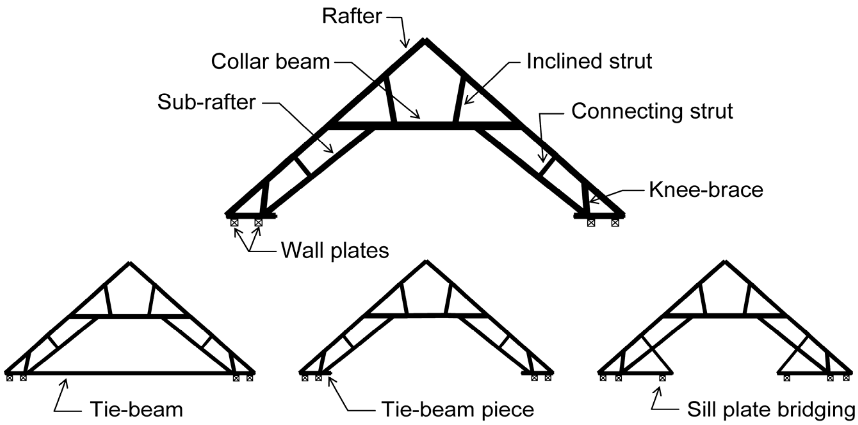


Figure 3. Generic configuration and variants

The reference configuration chosen is based on the general geometry and different elements' sections of the Noirlac timberwork, for which we possessed detailed survey drawings, including connections (Bontemps 2002). The frame spans 11 metres for a height of 4.9 metres, and is composed of main rafters connected in their upper part to the collar beam by inclined struts. Slightly inclined knee braces connect the lower part of the rafter to the lower horizontal piece. Finally, a small strut connects the rafter to the sub-rafter at right angles.

In the present analysis the applied load is vertical and symmetric, including the dead-weight of the timber structure (700 kg/m^3), and a uniformly distributed load (0.637 kN/m) on the rafters, representing the roof covering and normal snow loads, according to the frame spacing.

The variants differ in their footing system: the first two configurations address the usual systems using either a tie-beam to connect the lower ends of the rafter, or a tie-beam piece connecting each rafter to its knee brace. In these cases, the frame rests on two pairs of wall plates. A third system is included to address the specific case of the secondary trusses of Hermonville church, in which the tie-beam piece and the small connecting strut extend inward to meet slightly beyond a sill plate bridging on which the tie-beam piece rests.

The comprehensive analysis of these different frames is achieved through a series of computations aimed at understanding the roles of the different elements depending on the presence or absence of the others. This paper extracts the most significant computations from the process.

TRUSS WITH TIE BEAM

The structural behaviour of the basic rafter-truss with tie-beam, collar beam and knee braces is well known **(Fig.4, upper part)**. Under vertical loading, the tie-beam, always in tension, prevents the lower ends of the rafters from slipping outward; the collar beam and the knee brace, in compression, act as stiffening elements to the rafter. The significant bending of the collar beam under its own weight is worth noting (M = 0.8 kN.m), even if its deflection is quite small (1.65 mm).

The sole introduction of the inclined elements in the upper part of the frame results in a reduction of the bending moment in the collar beam, as it is now suspended to the rafters by these new elements. This is not in itself an improvement, since the stresses and deflection of the rafter are increased, while the stresses and deflection of the collar beam are hardly problematic.

Things are quite different when these inclined struts are introduced in combination with the sub-rafter **(Fig.4 lower part)**. In this case indeed, the sub-rafters work in compression as intermediate supports to the collar beam which, in turn, is stiff enough to provide efficient support to the inclined struts. These are now in compression, and allow a substantial reduction of the bending moment in the rafters. Indeed, the maximum being moment in the rafter drops from a value of 0.5 kN.m to 0.27 kN.m. At the same time, the deflection of the upper part of the rafter is halved. Another interesting point is the sharing of the normal compression force between the rafter and the sub-rafter.

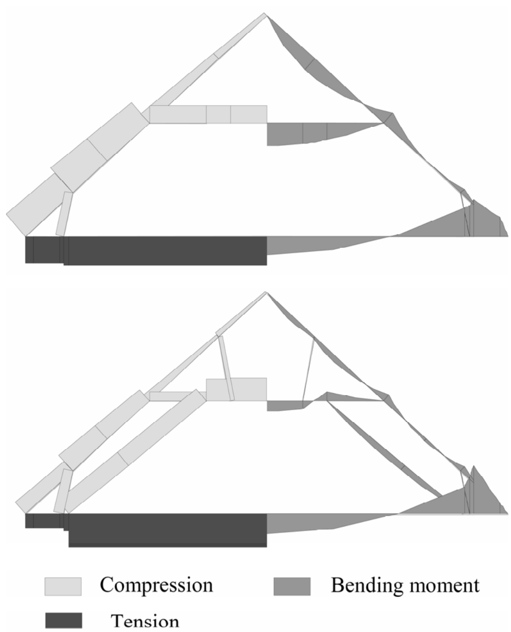


Figure 4. Effect of sub-rafter in combination with inclined struts

Finally, if the small "connecting strut" (or tie) is introduced between the rafter to the sub-rafter, changes are not spectacular: the bending and sagging of the sub-rafter are simply reduced to the disadvantage of the rafter from which it partially hangs **(Fig.5)**.

As a conclusion to this first analysis, it is clear that when the rafter-truss is equipped with a tie-beam, then the sub-rafter acts primarily as a support to the collar beam. Unlike what is generally thought, its role is not to relieve the bending of the rafter's lower part: on the contrary that bending increases due to the connecting strut.

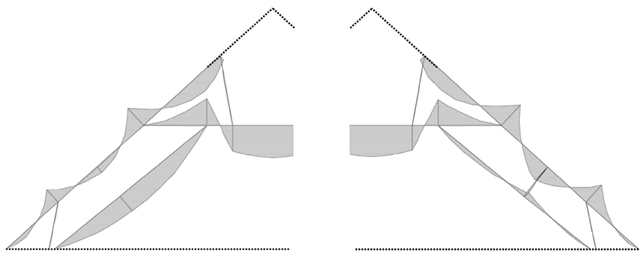


Figure 5. Effect of the connecting strut on the bending moment diagrams

It is rather the combined action of the sub-rafter and the upper inclined struts which allow both a moment and sag reduction of the upper part of the rafter, and a reduction in the normal force of the lower rafter segment.

Although the particular geometry of Noirlac is quite favourable to this mechanism, since the sub-rafter connects the collar beam close to the inclined strut, it still works the same way when the sub-rafter is much closer to the rafter such as in the Hermonville configuration. Simply in that case, the bending moment in the collar beam is higher, and the sub-rafter takes quite more normal force than the rafter itself **(Fig.6)**.

TRUSS WITHOUT TIE BEAM

In the plain rafter-truss without tie-beam **(Fig.7, upper part)**, the collar beam in tension resists the opening of the angle between the two rafters, while the knee brace in compression plays its part in stiffening the lower segment of the rafter.

Here the inclined elements introduced in the upper triangle to connect rafters and collar beam are rather disadvantageous if used alone. Instead of supporting the collar beam as before in the absence of sub-rafter, they are here in compression and transfer their own weight plus a small amount of force coming from the rafter to the collar beam, in which the bending moment is increased by a factor of five. The overall maximum bending moment in the rafter is also slightly increased.

The effect of introducing the sub-rafters alone is here spectacular **(Fig.7 lower part)**: the lateral and vertical displacements of the lower and upper ends of the rafters are divided respectively by 3.2 and 2.7. At the same time, the maximum bending moment in the frame shifts from the rafter to the collar beam.

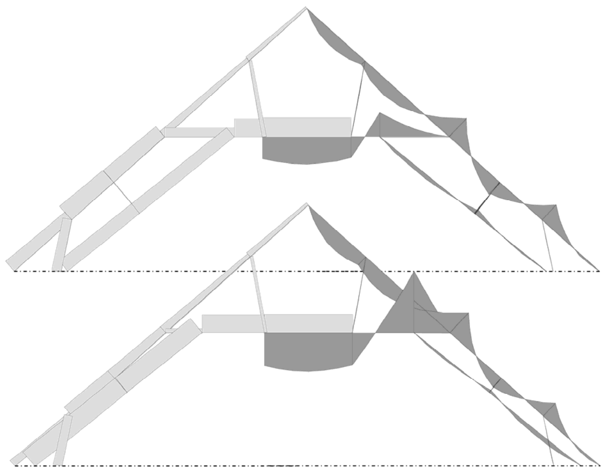


Figure 6. Effect of distance between rafter and sub-rafter

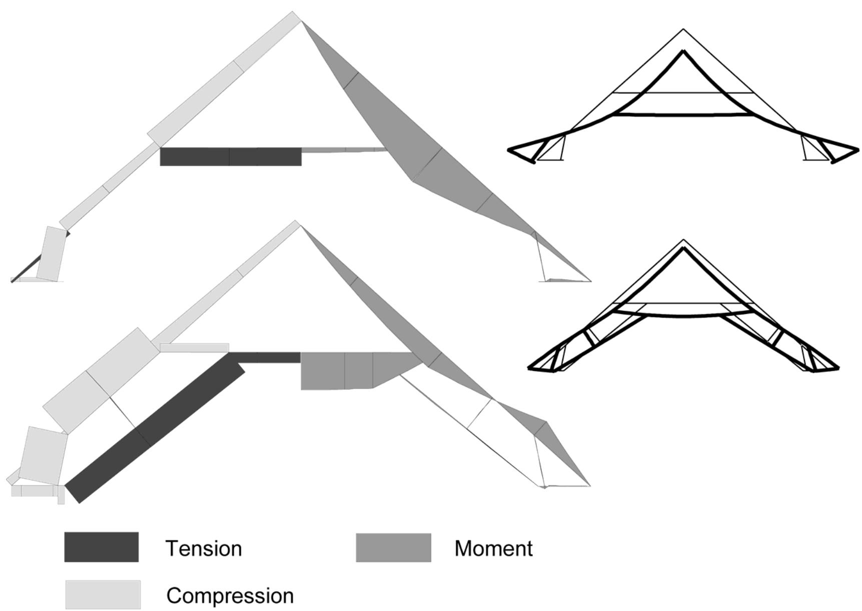


Figure 7. Effect of sub-rafter in rafter-truss without tie-beam

In this case, the mechanism is completely different from that of the frame with tie-beam. Here indeed the sub-rafter and central part of collar beam work in tension, thus playing the part of the suppressed tie-beam, restraining the tendency of the rafter feet to move outward. The horizontal component of the sub-rafter normal tension is balanced by the tensile force in the central part of the collar beam, while the vertical component results in a strong bending of that same collar beam.

When the sub-rafters are used in combination with the upper inclined elements **(Fig.8)**, these suspend the collar beam, and thus allow a substantial reduction in its bending moment (33%). This is achieved at the cost of increasing the bending moment in the rafter so that rafter and collar beam are equally loaded in bending. Also the overall stiffening is slightly increased.

Finally, as can be seen in the normal force diagram of the lower part of **(Fig.7)**, the end portions of the collar beam are in compression, and thus act as an intermediate stiffening support to the rafter.

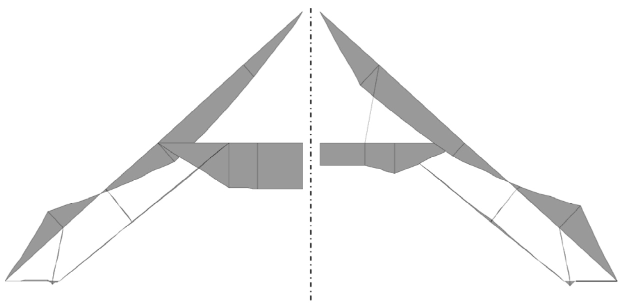


Figure 8. Bending moment with and without upper inclined elements

TRUSS WITH TIE-BEAM PIECE ON SILL PLATE BRIDGINGS

The secondary trusses of the Church of Hermonville in France, built around 1180, are similar to those at Noirlac, except that the tie-beam piece and the small connecting strut extend inward to meet slightly beyond a sill plate bridging on which the tie-beam rests. To understand the somewhat complex behaviour of the complete truss, let us first consider the case where the sub-rafter is suppressed **(Fig.9)**.

This configuration can be read as two rather rigid triangles ("enlarged footings") that carry the upper triangle. The connection between lower and upper triangles is provided by the continuous rafter. The collapse mechanism illustrated above **(Fig.9)** is prevented by the bending resistance of the rafter. This figure also shows that the upper inclined struts are ineffective, since their normal force is close to zero. As for the tension in the collar beam, it is 2.6 times less than in the plain rafter-truss without tie-beam.

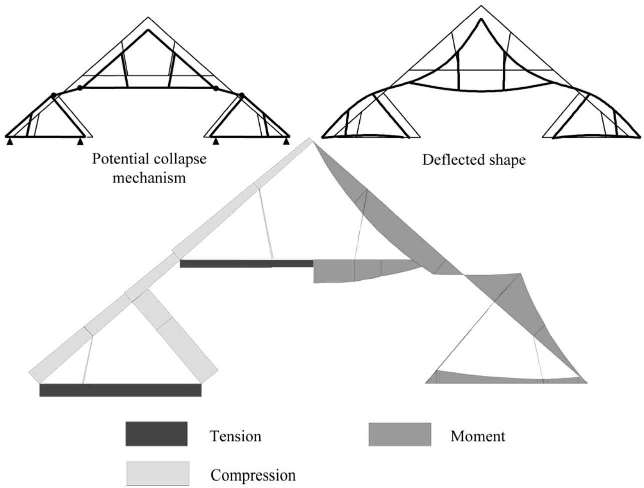


Figure 9. Mechanical behaviour in the absence of sub-rafters

The introduction of the sub-rafter has in this case has a similar effect to that obtained in the first variant with tie-beam. Indeed, it is in compression, and props up the collar beam so it can efficiently support the upper inclined strut which, in turn, relieves the bending moment in the upper segment of the rafter. But this compression in the sub-rafter can no longer be balanced by the tie-beam, and in its tendency to push the footings outward brings the lower part of the knee brace, rafter and exterior segment of collar beam into tension **(Fig.10)**.

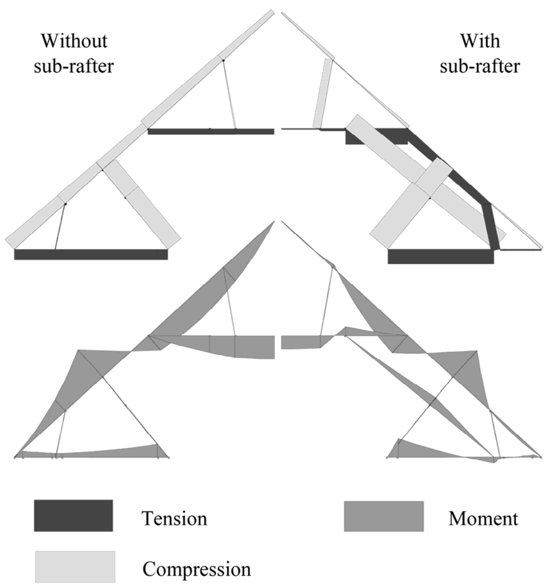


Figure 10. Effect of sub-rafter in rafter-truss with tie-beam piece on sill plate bridgings

SYNTHESIS

It is now clear that the role of the sub-rafter depends essentially on the "abutment" system used to prevent the opening of the rafter-truss. This system can take different forms, possibly qualified by their relative equivalent stiffness against the outward displacements of the lower ends of the rafter:

- A tie-beam, which in every truss provides a very high stiffness;
- A simple collar beam, with a quite low stiffness depending on the bending of the rafter beam;
- A collar beam and an "enlarged footing", which increases noticeably the equivalent stiffness;
- Wall plates in horizontal bending between the ties, located at the principal trusses. In this case, the stiffness depends on the proximity of the secondary truss to the principal.

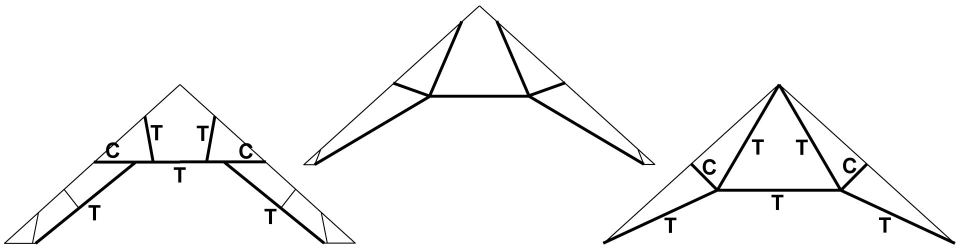
One can then study the effect of the sub-rafter on a basic rafter-truss without tie-beam in which the horizontal "abutment" system is modelled by a simple horizontal spring, the stiffness of which can be tuned to detect the point where the sub-rafter shifts from tension (low stiffness) to compression (higher stiffness).

It is found that in the generic configuration of Noirlac, this point could be representative of the stiffness provided by a pair of wall plates to a secondary truss located close to the centre of the interval formed by the principal trusses, if these had existed.

In no case does the sub-rafter work in bending sharing that of the rafter. Due to the presence of the knee brace, the rafter is much stiffer than the sub-rafter, so that the latter is rather suspended from the rafter through the connecting strut (tie).

Noirlac is the oldest French timberwork still standing we found that makes use of the sub-rafter. In that case, none of the rafter-trusses have a tie-beam, and so the sub-rafter works in tension, suspended from the collar beam, to provide horizontal restraint to the rafter footings. The connections used are indeed fit to work in tension, as argued by LePort (1988). It is tempting to draw a parallel with the Polonceau system (re?)invented in the mid-nineteenth century **(Fig.11)**.

Some 30 years later, at the end of the twelfth century, the carpenters of Hermonville church, St John's Hospital chapel and St. Peters cathedral in Lisieux all used the sub-rafter in combination with a tie-beam. On the one hand, it can be noted that the sub-rafter is quite close to the rafter so that it is less efficient to strengthen the collar beam onto which diagonal struts transfer some load from the upper part of the rafter. In the case of Hermonville, the assemblies for these pieces are indeed fit for compression and not for tension (mortice and tenon). On the other hand the sub-rafter is also used in the secondary rafter-trusses where the absence of tie-beam also reduces its efficiency, eventually to zero.



Noirlac 1160 Polonceau 1840

Figure 11. Comparison between the Noirlac timber frame an Polonceau frame

It should be noted that, at the same time, corner braces appear in the obtuse angle between the collar beam and rafter in a good number of timber roof structures in northern France. These elements are also able to provide more efficient intermediate supports to the collar beam.

St. Martin Church in Laon (northern France), is a further example of the use of the sub-rafter in tension, very like at Noirlac, but some 75 years later.

Other later timberworks include elements that could be seen as variants of the sub-rafter, either parallel to the rafter and extending all the way up, or not parallel to the rafter, and generally extending to queen posts.

CONCLUSION

As far as the first known occurrence of the sub-rafter is concerned, it may be assumed that it served the requirements of structural behaviour and/or that it helped in the construction process (LePort 1988). If the idea was indeed to prevent the opening of the truss in the absence of a tie-beam, and this is compatible with the nature of the connections used (dovetail joints), why give it a direction parallel to the rafter? This could denote that the carpenters did not completely understand the mechanics of that tie, as a more slanted element would have been more efficient, or that other considerations were considered as more important: parallelism as a formal option, rather than concern for the collar beam bending...

The fact that the same sub-rafter is later used in combination with a tie-beam could lead to the idea that an initial concern for structural behaviour (if any) is lost, and that only the morphological concern is retained in the transmission of the carpenters' know-how. In any case, it is not unthinkable that carpenters may have then identified a new mechanical relevance for the sub-rafters, as supports to the collar beam, thus later changing its shape to a knee-brace, the better to discharge this structural duty.

Even though this conclusion cannot be made from the present study alone, other mechanical analyses concerning different structural elements (Fleury & Mouterde 2006) lead to the possibility of a similar kind of learning process, consisting of:

I. A first morphological innovation based on intuition for structural mechanics;
II. A copy of the formal proposition out of context, loosing the structural idea;
III. Reinterpretation of the mechanics in the new context;
IV. Morphological modifications, the better to match the new structural interpretation;
V. Eventually, the return to step (II).

At least two directions should be taken to go a step further concerning the status of the sub-rafter. The first is to study the effect of wind loads. Indeed, other studies show that the action of wind in the plane of the rafter-trusses is predominant, and could well be the principal concern for the carpenters of that time. In addressing this type of asymmetrical loading, structural intuition and intentions are more difficult to detect and formulate. The other direction would be to go deeper into the assumption that the sub-rafter may have served an important function during the erection process. This idea, proposed by LePort (1988), is not yet backed-up with an example of such a process.

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