Digital Stereotomy and Topological Transformations: Reasoning about Shape Building

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Stereotomy, as a historical subject, shows how to build stone/wood vaulted architectural systems of hard spatial complexity, where specific geometrical rules and correspondences set the relationship between system and part. Such a difficult, but useful knowledge for the *tailleur de pierre* (stone cutter) can be made easily accessible by the experts, thanks to the evolution of the existing three-dimensional modelling software that allows the shape-planning and shape-building process to be checked unambiguously. This essay aims to show the capabilities of computer- modelling techniques when applied on classical stereotomy studies, and to analyze the topological transformation techniques applied on freestone architecture elements.

Our working hypothesis is to compare two computer-modelling techniques: direct and indirect modelling, both used to study the stereotomic shape. In the first case the shape rises by consequential protrusion processes and Boolean works on two-dimensional shapes coming from *trait géométriques*; in the second case the shape rises by modelling processes based on topological principles, that means using volumetric transformation and deformation tools. Indirect modelling enables very complex stereometric shapes to be built and checked through hard conceptual work and easy practical operations. The geometrical conformation of every single ashlar, of a specific vaulted system, will be the result of an appropriate series of simple solids geometrical transformations, producing a topological correspondence.

The proposed studies will result in some topological changes of the famous stereotomic system known as "flat vault"; the particular flat bond will be changed into different spatial geometrical systems in order to show this method's advisability and convenience. The information gained from this research will be useful in the analysis and projecting tools for the freestone architecture.

THE NOTION OF STEREOTOMY

The word Stereotomy or Cutting Solids firstly appeared in 1644 in the Jaques Curabelle's libel case against Desargues: *Examen des oeuvres du Sieur Desargues.*

> Par grace & privilegi du roy il est permis à Iacques Curabelle, de faire imprimer, & vendre par tel Imprimeur & Libraire que bon luy semblera, un Cours d'Architecture par luy composè, diuise en quatre Tomes; Le premier desquels contient, La Stereotomie, ou

> Section des solides, appliquée a la coupe des pierres, Et son appendix des quadrans, tant par rayon d’incidence, que de fraction & reflexion…. Paris, le 4 decembre 1644.
>
> (1643, Extraict du privilege du Roy)

(With permission from the King, Jacques Curabelle is allowed to print and sell with the favourite editor and bookseller, a Course of Architecture, of his own writing, divided into four volumes; the first contains Stereotomy, or section of solids, applied to stone cutting, and its supplement, of solar watches of incident ray as well as of fraction and reflection.)

Probably taken from the union of two Greek words (□□□□□□’ solid e □□□□’ cut), stereotomy represented, according to Jacques Curabelle the cultured abstraction of something handed down through the centuries as “ the art of the geometrical line”. Although the method and definition of *art du trait géométrique* dates back to about a century earlier, when Philibert de l’Orme, in his printed work of 1567, reported his authorship of this new method: *I’en trouuay le traict & inuentay l’artifice en ladicte année mil cinq cens trentefix, par le moyen & ayde de Geometrie, & grand trauail d’efprit…* (I have discovered the *trait* and created the method in 1536, thanks to geometry and to a big care…)

Due to the debate between Desargues and Curabelle on the relative importance of theory and practice, the legitimacy of the method and, consequently, the status of their support was called into question. Curabelle argued that the feasibility criterion was of primary importance while Desargues thought that it was the exactitude of the geometrical thought that was more important. Actually, stereotomy is the legitimate daughter of theory as well as of practice, as both are important in architecture, and, through Philibert de l’Orme’s speculation, the art of stone cutting got a universally recognizable value. It is therefore Philibert de L’Orme who is attributed with the origin of the stereotomic discipline and the subject, first described in the third and fourth book of *Le premier tome de l’architecture,* developed continuously until the middle of the 18th century.

We think of the birth and evolution of the art of *trait géométrique* as an ongoing issue, and, as was said before, we think of it as a reachable outcome aimed at bringing together building site experience with the development of projective techniques close to the current perspective codification. Jean-Marie Pérouse de Montclos, in his studies on the French Renaissance, wonders what could have been the scientific and geometric knowledge of masonic masters who preceded de l’Orme in codifying sterotomy as *art du trait géométrique.* De l’Orme himself declared that his geometry came from Euclidean geometry:

> [It was] studied, learnedly interpreted and documented, presented and highlighted by Mr Francois de Candale, and publicly read and exposed in French language by the king’s masters: Mr De la Ramée, Mr. Cherpentier and Mr. Forcandel in this university of Paris.”
>
> (1567, Book III, p. 116)

An oral tradition is supposed to exist, preceding the architectural treatises, which referred to Euclid (of the *Elements* and *Optics*), not to Vitruvius. This could partially explain de l'Orme's aim of "methodically setting", according to Euclidean logic, the art of *trait géométrique* of the medieval masonic culture, and his determination to connect it to Vitruvian architectonic speculation. Persuaded that the medieval builders' skill in making is the only opportunity for the Renaissance architect, in his treatise de l'Orme commits himself to unify *trait* practice with Euclidean theory ("*conjoindre la pratique des traicts avec la théorique d'Euclide"* - fol.62), devoting himself to a review of Euclid "… *revoir Euclide* …". Method, indeed, seems to be the biggest concern for de l'Orme, who swears to drive Vitruvius to Euclid, i.e. to take him to a reliable method ("… *reduisant à une certaine méthode…"*), or, asking master masons to summarize Vitruvius in a methodical and ordered way ("*…vouloir reduire Vitruve en bon ordre & methode…"* - fol. 62).

Our hypothesis does not seek a direct geometrical connection between Euclid and de l'Orme, rather, we want to focus on the interest he had in Euclid's work, considering it a very high digest of methodological rigour. Thus, it seems interesting to quote Robin Evans who postulates:

> Certainly there are two geometries vying with each other in de Delorme's work, but I would categorize them as Platonic and projective rather than Euclidean and Vitruvian […] throughout the 13 books of the Elements there is not a single reference to, or utilization of, projection, whereas projection is fundamental to the traits.
>
> (Evans 2000, p. 200)

The method of *trait géométrique* is supposed to be inspired by Euclid, and projective geometry is supposed to have developed from the *trait*, which cannot be connected, neither to one nor the other, being in an intermediate position. The analytical demonstration of the fact that the *trait géométrique* comes from Euclidean geometry, and the demonstration of how building techniques have reached, thanks to de l'Orme, an Euclidean methodology, surely needs an independent treatise, much more complex than the present one, which could expose, within its limitations, the theories considered most important.

As everybody knows, the axiomatic-deductive Euclidean method begins with the intuitive definition of very easy concepts and then, gradually, establishes a wide corpus of results, connected to one another. It creates a strong and rigorous building that makes all the processes perceptive, comprehensible and intelligible. In the same way, the structure of stereotomic treatises, from de l'Orme's onward, is organized so that the *trait géométrique*, as it grows complex, follows the easier route. Methodological description of building stages, both of *epure* (geometrical line in full scale (1:1)) and *tout court* of architectonic element follows a consequential criterion that allows the stone cutter easily to reach the building process demonstration. Finally, the ruler and the calliper are the geometrical building tools assumed from the start, both in Euclidean Elements and in Stereotomic treatises. Therefore, stereotomic fabrication tools are those Masonic symbols that the applied knowledge of the l'Orme refers to.

By carefully analysing the principles of the stereotomic discipline, applied to stone cutting in order to create architectural elements and/or systems, we noticed that stereotomy is regulated by three distinctive and “invariant principles” of this disciplinary corpus.

They are:

- *Pre-figurative invariant*: that is, the subdivision capacity in appropriate sections of a vaulted system
- *Technical/ geometric invariant*: that is, the capacity of geometric, punctual definition of an architectural system and of ashlars and its realization. (projective technique and cutting technique)
- *Static invariant*: that is, the capacity of providing static balance of the architectural system through dry-stone jointing (graphic and mechanic static of rigid structures).

According to these three parameters, capable of being variously ordered, we can define a selective filter, in order to discriminate between the general stone architectures and the stereotomic ones, which have to be intended as highly speculative forms of stone architecture. Even the expressive quality of a vaulted system, where the decorative part intrinsically fits together with the structure, can refer to the first of the three invariants. The second invariant can represent the distinctive aspect of a stereotomic work, where the technically realizable capacity plays the main abstract part in the planning stage. The third invariant can be linked with an aspect of neoclassical criticism against stereotomy, that which they taxed with anti- classicism. Thus the desire of dissimulating the value of Vitruvian *firmitas* drove stereotomy to an excess of static virtuosity, seemingly defying the laws of gravity, turning out odd shaped planning. One of the more interesting of Charles Perrault’s definitions about that, holds that stereotomy is the art of “ using the same stone’s weight against its gravity to support it on the top, making full use of the force that would bring it down.” (Pérouse de Montclos 1982, p. 85).

In conclusion, the “three invariant principles” are strictly connected within a sterotomic work where geometry, decoration and technical control of the building process are the basis for a lively and fruitful planning speculation.

THE TOPOLOGICAL IDEA OF STEREOTOMY

Instead of forming a specific bond, the ashlars of a sterotomic vaulted system can be very complex solids, a sort of envelope and assembled surfaces. Indeed, the role of the *trait géométrique* is to describe and draw on the plane of such surfaces. It is, however, not always possible to develop surfaces. For instance, we cannot transfer onto the plane, with their real shape, either rotational surfaces, or their results, neither in general, double curved surfaces. Stereotomy allows us to describe and to develop on the plane even double curved surfaces, with appropriate approximations.

A cupola can be thought of as a not-continuous surface, but created as an ensemble of conic surfaces within parallels, as well as an ensemble of cylindrical surfaces within meridians. Thus, to turn a spherical surface into a cupola or into a Bohemian vault of stereotomic nature, we must section it in cones or cylinders, or in any developable surface.**(fig. 1)**

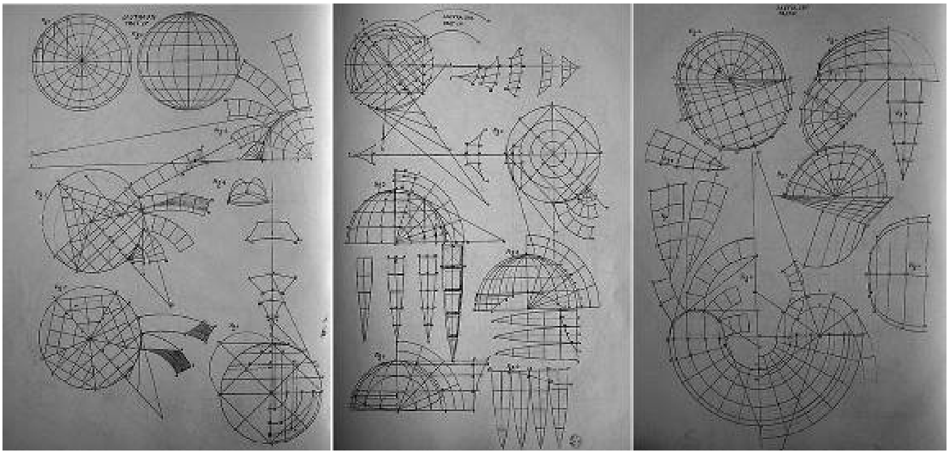

Figure 1. Surfaces development: (Guarini G., Lastre XII, XIII, XIV, trat. IV).

The analysis of specific treatises shows that, from Delorme to Vandelvira, until the 19th century, such a technique represented, in France and in Spain, the unquestioned means by which to design and to build most of the *vaulted* and *domed* stone and wood spaces. The development of a surface is necessary to their physical construction. Cutting the ashlars of a vaulted system by using "*panneaux*" (development boards) and "*biveaux*" (moulds) technically lies in creating flexible panels, which represent the surface development on a plane. They are folded on the appropriately worked bed-stone in order to transfer on it the outline of the ashlar surface. Moulds are used to control the angle between ashlar faces.

The concept of "folding" is a typical topological concept according to which a flat paper and the same paper in cylindrical shape are equivalent, topologically speaking. Thus we get a homeomorphism between the flat rectangle and the cylinder. If we keep on folding the cylinder we get an anchor ring surface which is topologically equivalent to the cylinder, and, consequently, to the starting rectangle. So we get a one to one correspondence between a flat surface and a spatial one. Similarly, to build a cupola we have to construct flat panels that create the one to one correspondence with it. Obviously we are speaking about conceptual analogies between topological and stereotomic processes, as topology is the study of the properties of geometrical structures which do not change when the structures are subjected to continuous deformations that mean a transformation of the figure, where jerks or tears are not allowed. So, if we assume that topologically *continuity* is the only equivalence between figures, we have to underline the different mathematical nature as for a stereotomic process. Indeed, as we have seen, it is characterised by discontinuity and segmentation of the surface. Topology, or, according to H. Poincaré, "geometry of elastic figures", can be conceivable with the sense of touch. In fact, if we imagine the physical model of a topological object, we can draw a hand over it without finding interruptions.

A stereotomic system, both a vault and a cupola, is obtained by sectioning the continuous surface in a number of flat polygons which, according to the number of divisions, can show the perception of the exact curvature. Among stonecutters this technique is also known as the "*douelle plate*" method. *Doele* or *douelle*, from the Latin, *Dolium*, or cask; the stave of a cask metaphorically means the vault intrados. The flat surface passing through the arch chord of a *doele* is called, *doele plate,* and represents a preparatory phase in order to define a vaulted space. The number of polygons which make acceptable the *plugging* of a curve surface, is the variable which allow us to speak of continuity, and therefore, of stereotomy in a topological sense.

These facts determine the developable surface quality closer to the abstracted surface of a vaulted space. The interest as well as the value of topology depends on the immediately evident content of its problems and principal rules. These give us immediately the sense of space, first as an area where continuous shape changes take place. These are qualitative aspects proper of those *figures* that remain invariant as regards particular changes. Topology deals with geometrical issues that, to be studied, do not even need Euclidean concepts of line and plane, but only need the existence of a continuous connection between the points of a figure. Euclidean geometry is based on concepts of distance and measurability, anthropologically speaking more *natural*: we estimate distances by eye and this possibility makes the concept of measurement extremely natural to us.

In the history of geometry, topological problems emerged later than the projective ones, in fact only in the 18th century, in the wake of the earliest formulation of non-Euclidean geometry. The first work relating to topology is attributed to Euler who, in 1736, published an article about solutions for the Königberg bridges entitled, *Solutio problematis ad geometriam situs pertinentis* (The solution of a problem relating to the geometry of a position). He gave a mathematical demonstration of the impossibility of finding such a route. That study indicates that Eulero was theorizing a new kind of geometry where the parameter of distance was not relevant. Eulero's researches cleared the way for following mathematics: i.e. Gauss, J.B. Listing, Poincaré, Brouwer, who drove topology into many other scientific disciplines.

The best known and important "physical objects" in topology are the so-called: "Möbius Band" and "Klein Bottle": both due to nineteenth century mathematics. They represent all the characteristics of topological surfaces as absolutely continuous surfaces, without any interruption, coming from transformation and deformation processes of a flat starting surface. It seems interesting to notice some morphological analogies between these objects and some stereotomical elements such as the stringer of helicoidal stairs called "*limon*" in French technical literature. From our point of view morphological analysis, oriented towards the relationships between topology and stereotomy, lies in the comparison between families of *similar shapes* more than in the description of individual cases.

According to this key to the reading, topologically those shapes due to appropriate processes or geometrical deformations of other shapes (such as arches/ oblique arches/flying buttresses, flat vault/tunnel vault/anchor ring tunnel vault/ helicoidally tunnel vault, tunnel vault/conic vault/trompe… and so on) are *similar stereotomic elements*. Obviously other kinds of topological correspondence in stereotomic systems are related to already known surfaces developed with the corresponding spatial surfaces. In this last case, as we shall see later, we have to focus on the importance of the method when the vaulted system gets an important decorative system, or, if the same bond shows a complex decoration.

HYPOTHESIS AND MEANS OF SPECULATION

The object of our speculation is the *procedure* which consists of deforming a stereotomic system and studying its consequential alteration. The new stereotomic system arrived at represents the result we got submitting the other system (that may be called “primitive”) to a homogenous deformation, function of a specific variation. In other words we shall be able to understand the morphology of a *complex* stereotomic system supposing it proceeds from the appropriate processing of a *simple* stereotomic system.

It is useful to remember that our study is more effective if applied to those stone/wood stereotomic systems having a rich decorative texture coinciding with the structural one **(figs. 2 and 3)**

Analytical tools are only information system- based. That is why there is a strong conceptual and operative affinity between stereotomy and solid modelling, which allows the terms of the demonstration to be changed, from *physical* to *virtual*, returning in a singular updating to our speculation. The dematerialization process of the peculiar phases in the stereotomic construction process, free from physical/material links, allows a bigger freedom for the analysis of: space, shape, volumetric composition, de-composition in parts, bond, decoration, building, statics and the mechanics of rigid bodies.

Anyway, it must be remembered that with the deforming operation on pc, because of the huge potential of the instrument, it is always possible to loose the formal control that could easily mean ending up in generic and uncontrolled morphing operations or ruinous metamorphosis, far from the constructive thought of the shape. The study of the shape can be simply descriptive or qualitative and can become analytical or quantitative. Mathematics that allows the number abstraction to regain possession of geometric figures becomes *computer graphics* or *info graphics*. As everybody knows, most of the computer graphics software that we use today has been created for other sectors such as the car industry, aeronautics, or cartoon cinema. Only recently have they been used in the architectural research to help with the study and the planning of three-dimensional models. A computer-graphic model wants to represent a real object mathematically formalized. It is a *mathematical* image of the real object. This image, according to specific analytical needs, can be more or less detailed and more or less abstract.

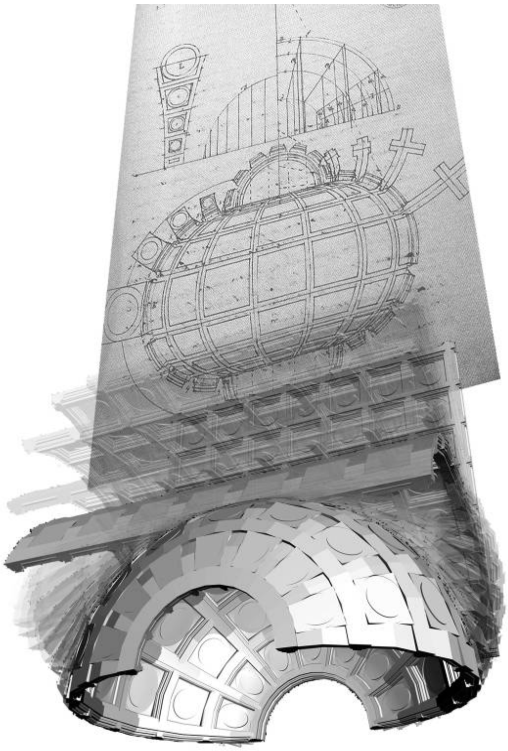

Figure 2. Transformations: de Vandelvira's cupola (1575-1580): *Boveda de Murcia*.

Depending on the mathematical formalization, we shall get an appropriate formal correspondence with the real object that we are re-creating as a model, which remains an ideal interpretation of reality. Therefore, to make things easier, there are kinds of mathematics aimed at guaranteeing a representation as close as possible to the real object, and mathematics that recreate reality more or less approximately. Today modelling techniques refer to two main methods that can be distinguished according to the mats used. They are: “discrete entity”, that is polygonal models, and “continuous entity”, that is NURBS models (Non uniform Rational Bézier –Spline).

We have to remember the main difference between the polygonal model and the NURBS. The first gives back a discrete geometry of the real object the second gives back a continuous geometry. In order to optimize and make compatible these modelling procedures it is possible to convert NURBS into polygonal models and vice versa.

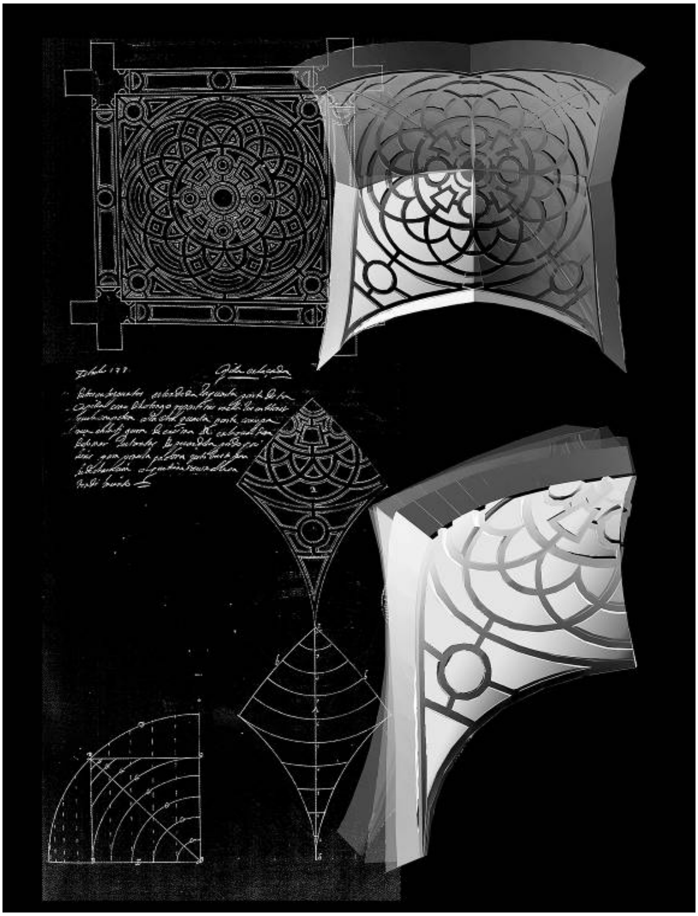

Figure 3. Transformations: decorated Bohemian vault (de Vandelvira (1575-1580)), . *Capilla enlazada*).

Among modelling mats, our study should focus on a kind of parametric modelling based on the theory of modifier-based modelling. We have to consider the modelling process as a *flexible sculpture* of digital three-dimensional data: we start with a simple shape, such as a parallelepiped or a sphere or a whole of common solid volumes and change them through “parametrical topological deformations” to get the complexity standards proper of the original model. We shall reach the model indirectly, just working with appropriate deformations and not directly through classic modelling, which would be very hard in three-dimensional modelling in the case of very complex objects. Programming has foreseen the introduction of a new approach to the computer–graphic modelling, based on deformation tools, so as to give indirectly a shape to the digital object. These instruments, actually mathematical algorithms, are perfectly suited to the characteristics of planimetric modelling, so that expressing numerical values of parameters makes it easy to follow the results of the given deformation.

The most important modifiers in modelling are those that operate on the object, or on a part of it, an easy deformation according to a directional axis. The Bend modifier, for example, deforms the geometry of an object, bending its rectilinear development. Such deformations are possible thanks to the distribution of segments that configure the polygons of "plugging" of surface on the surface itself: every intersection between vertexes acts like a joint. Thus the more segments on the surface the more it will be "change proof". Once we intuitively understand the deformation processes that brought us to the final configuration of a stereotomic system, we can go backwards, modelling an appropriate solid and then changing or deforming it in several ways, giving even more than one modifier to the same object. Among the most powerful modifiers we find: Taper, Allunga, Twist, Non Uniform Scaling… and so on. Through this expanding process of modifications the whole drawing gets a new configuration with new shapes and dimensions. CAD parametric system automatically recreates the whole model, rebuilding the new geometry, correspondent to the user's requirement that is obtained by using modelling processes which are actually easier than the classic modelling that could be useless for very complex spatial cases **(figs. 4 and 5).**

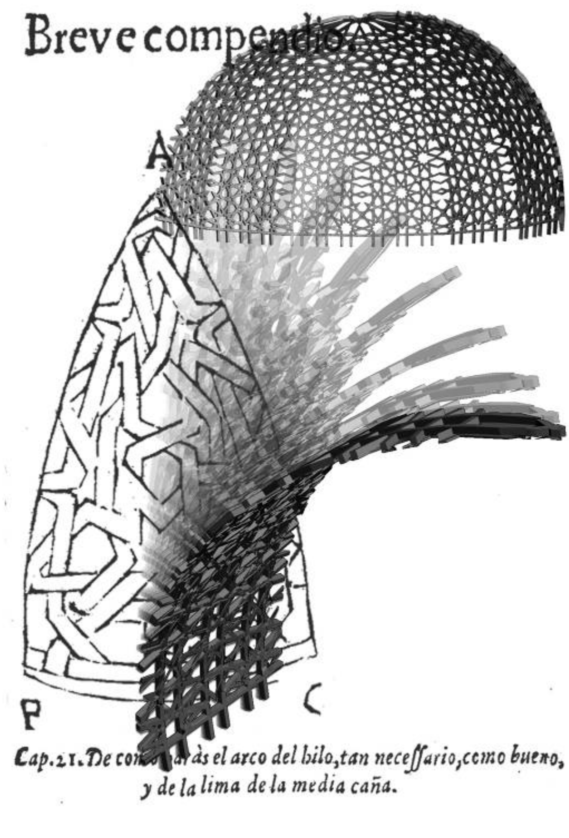

Figure 4. Transformations: cylindrical development and its cupola (López de Arenas, D., 1727. *Carpintería de lo blanco*) *media naranja*, *Salón de Embajadores de los Reales Alcázares*, Seville.

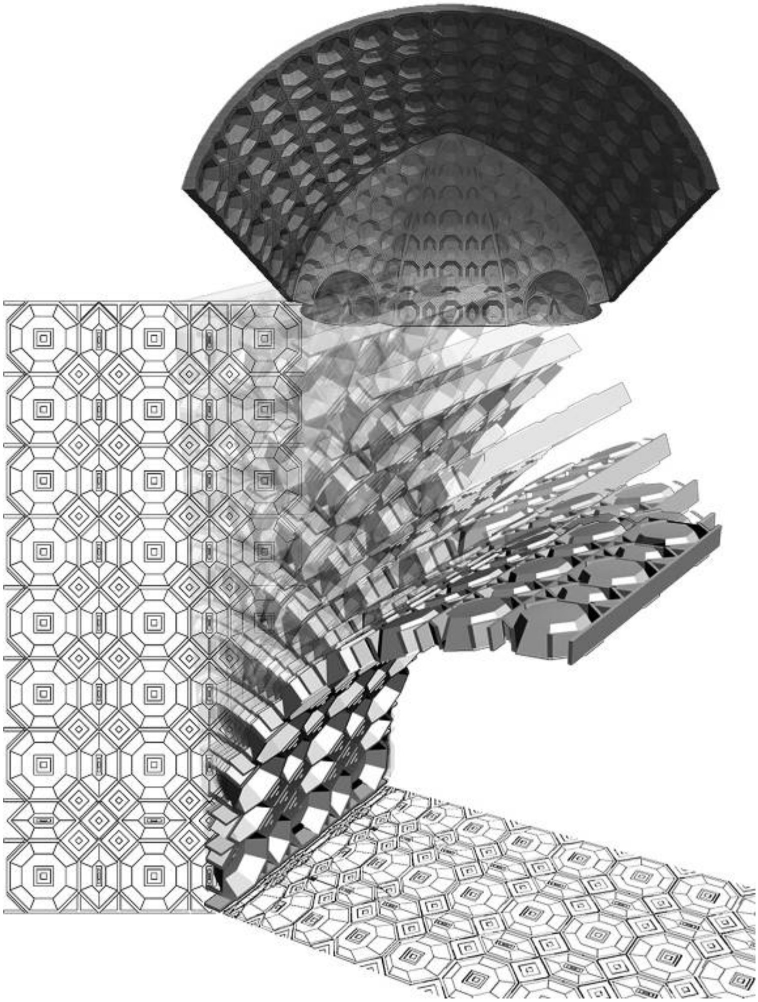

Figure 5. Transformations: Coffered ceiling of *media caña* covering one of the first floor halls of *los Reales Alcázares,* Seville.

CASE STUDY: TOPOLOGICAL VARIATION OF THE FLAT VAULT

The stereotomic system known as "*voute plate*" represents one of the most interesting technical/ stylistic speculations of the stone cutting art applied to building, though rarely employed in history. As we shall see this kind of stereotomic system is suitable for the application of the exposed techniques. The term, "Flat vault" is an oxymoron, linking two geometrically contradictory words: the *vault,* which is a three-dimensional surface by nature, and the word *flat,* referring to a two-dimensional surface.

The historical problem is to find a building solution to cover a space with a flat floor constituted by discreet elements: in other words, to build a vault with zero radius stone-ashlar.

Such an issue, free from functional influences, lies in the stereotomic speculation area of many French and Spanish treatises (Frezier, A.F. 1737, Rondelet, J.B. 1802, Douliot, J.P. 1825, Rovina y Rabassa 1987). The static principle which this covering system relies on, presumes that the overhanging charges way goes from the vertical to the horizontal, passing through the scarf of every single ashlar of the vault, which seems totally compressed. Like the analogy between surface and plane, the same principle refers to the static functioning of the straight arch, which can be assumed as a model of one of the possible variants of the flat vault. Such a stereotomic system can be constructed, ideally, by *extrusion* of the straight arch (in case of squared or rectangular systems) as well as *rotation* of a straight arch (in case of circular systems). This earlier division envisages every possible geometry and bond of the straight arch. The extention on a plane of the straight arch produces a flat vault presenting a variable number of different ashlars, symmetric to the axis of symmetry of the straight arch itself.

Not to fall into this trap, and in order to prepare a flat vault with just one appropriately configured stone ashlar, brings into play the real geometric speculation we are truly interested in. The first patent proposing such a constructive solution, comes from a French engineer from Marseille, Joseph Abeille (1669-1752). The patent was published in 1699 in *Machines et inventions approuvées par l'Académie Royale des Sciences.*. The ashlar-standard repeated in series so as to realize this flat vault is a polyhedron having an axial section in the shape of an isosceles trapezium. The static functioning of this solution is one of a flat bi-directional plate working identically in both directions, where every ashlar supports and is supported so that the vault comes into play just when the assembling process is finished. Abeille's flat vault does not have interesting practical advantages but, compared with the above-mentioned systems, where all the ashlars are different, it is optimized for being just one shape of standard ashlar. The unique geometrical conformation of an ashlar guarantees the mutual support of the blocks within the vaulted system. Abeille's flat vault presents two visible surfaces once the assembling is ended: a squared homogeneous and unique net on the extrados, and the other reminding one of the motif of the textile work with pyramidal holes on the intrados. Such an intrinsic characteristic of structure makes the system particularly suitable for realizing a specific ceiling with a peculiar "coffered structure" in its intrados and a squared, regular floor on the extrados. The interruption constituted by pyramidal holes, even if *decorative*, can be considered a drawback of Abeille's solution.

The 31st *planche,* page 77 of the second volume of Frézier's treatise (1737-1739) is a marvellous summary about flat vault building systems. Here it is possible to find the several geometrical shapes of ashlars operating possible variations on the theme. At the top of the page is represented a straight arch flanked by two apparently unsuitable schemes. They represent the possibilities of covering squared or circular areas with wooden beams in the shape of fylfot (a swastika turned counterclockwise), smaller than the spans themselves. Actually these schemes perfectly capture the static principle upon which the flat vault premise is based. According to it every ashlar, like a wooden beam, "supports and is supported." According to Frézier it is evident that Abeille's solution was inspired by the famed ceiling of Sebastiano Serlio, whose aim was covering big spans with small beams. The connection between the "lithic" system of flat vaults and wooden carpentry survives within building theory until 1856, with two bi-directional solutions illustrated in A.R. Emy's "Treatise on the Carpenter's Art".

In the centre of the page Frézier shows a vertical section, two planimetric views (from above and from below), and axonometrics of single ashlars of a flat vault, built up with four different ashlars – a sort of inventory of three-dimensional ready- to- use patterns. Three of these four solutions show ashlars made of flat-faced polyhedrons, the other shows the ashlar made of an envelope of flat and lined surfaces. The latter is perhaps the most interesting configuration from the speculative point of view, as it represents the polished solution developed in 1704 by the Carmelite Jean Truchet, known as father Sebastien (1657-1729). We shall return to this later in the essay in order to solve the problem of Abeille's theory of pyramidal holes on the estrados. Frézier solves that problem using two variants of ashlar representing an *adjustment* and *canonization* of Truchet's solution: in the first case there is no envelope of lined surfaces, but an envelope of appropriately connected flat faces; in the second case the lined surfaces become conic-developable surfaces.

Coming back to Truchet's theory as stated in "Mémoire concernant les voutes plates", in *Recueil de l'Académie Royale des Sciences (1704)*. Our interest in this theory focuses on the capability of infinite formal generalization of a simple geometrical body proper of the "three-dimensional pattern" system. It is important to remember that father Sebastien Truchet was a mathematician, and his name is linked mainly to the "pavages de Truchet", even though he is renowned for having taken out numerous patents (from the unit point, to the typographical character known as *Romain du roi,* until the Truchet's Paraboloid). In his *Pavage*, Truchet studies how to combine simple shapes in order to get nice decorative drawings, such as the black and white two-tone square divided by a diagonal. Moreover, in 1704, he publishes the results of his researches in *Comptes-rendus de l'Académie des Sciences*. The *tiling* theory that, surely, must have been the source of inspiration for the ashlar of the flat vault, was not new, as we find the same issue dealt with in Emile Fourrey's, *Curiosités géométriques*, but Truchet was the first to publish detailed studies about all combinations of the *tiling system*.

Conceptually, a *tiling system* is a standard surface created by connecting simple elements without holes and superposition, using a small quantity of different elements. Thanks to this central principle it is possible to create complex decorative patterns, especially with complex and differently coloured basic elements (The exercise refers to a competition organized by the Gutemberg Association). The problem have been discovered, exposed and first studied by Philippe Esperet, teacher of mathematics in the Lycée Henri IV, in Paris, in 1995.

> In 1704 Dominican, father Sébastien Truchet, interested in maths as well as in arts, published a work where he showed that an infinity of motives could be produced by the simple connection of half coloured squares. Indeed in his work appeared the earliest expression of combinatory theory principles and of crystals symmetry. Actually this work represented the origin of another work of the even […] Dominican father Douat who published in 1722 a book full of a big quantity of superimposed basic motive drawings. This book had a big influence on all European decorative art during the 18th century.
>
> (Smith 1987, pp. 373-385)

It is important to establish a connection between the theory behind the "Truchet's pavages", and the flat vault "Truchet-ashlar" theory in order to capture the mutual peculiarities, that is, the capability of creating several decorative patterns covering one surface. It transpires that the "Truchet's pavages" system can become three-dimensional if it is appropriately connected with the flat-vault concept. Both the patents can be co-related so that they can create infinite decorative variations of the flat vault, where decoration and structure completely fit together.

History thus gave us two techniques that allow us to:

- Build a *pushing* ceiling, made of similar, appropriately scarfed stone elements.
- Infinitely vary the shape of stone elements and, consequently, the whole ceiling decoration.

If we put aside the ceiling and focus on the two above mentioned considerations, referring to a solid (parallelepiped with a small thickness if compared to length and height) made by assembling several blocks appropriately configured, which guarantee a strong link, we can go on making some important reflections, linking stereotomy with topology. So, we are back to the starting point of this study, where the interest is in manipulating, in a creative way, the shape of structures, historically codified, in order to stimulate new speculation both in analysis and planning. Bending technique deformation, explained above (because of which from a rectilinear segment we get an arch in circumference, or, from a plane we get a cylinder or a sphere) allows us to formalize the stereotomic vault made by the scarfing of the flat vault ashlars. This work is aimed at solving some of the most important problems connected with the creation of vaulted spaces. It seeks to:

1. Improve the mutual anchorage and friction of the ashlars of a vaulted system.
2. Reduce the masonry gauge
3. Conceive of the creation of a vault both as a structural and decorative system.
4. Easily and constantly vary the shape of a bond
5. Rationalize the number of standard ashlars to build any vaulted system.
6. Get automatically all the geometrical data for both the manual and electronic cutting of any standard ashlar. (cad/cam/cnc)
7. Provide automatically all the number data useful to the structural calculation.

Experimentation have been conducted on both Abeille's and Truchet's flat vaulted patents. They have been subjected to a *bending deformation* that changed the plane into a semi-cylinder. We then proceeded to build a barrel vault with the mentioned systems (**figs. 6, 7 and 8**). All proposed pictures have been conceived in order to contain the dynamic effect of variation within a static image. Due to the limited extension of this work only the bending deformation has been presented, as we consider redundant the description of other modifiers, where the usual rule about the infinite chances for making variations and metamorphoses apply. In order to change the flat vault into a barrel vault we have had to find a field of geometrical/formal variation of the vault itself, correcting modification parameters. Attributing a single curvature to the vaulted system meant obtaining two standard ashlars for the realization of the whole vault. This number increases together with the increase in the curvature degree of the vaulted system. To understand the possible formal generalization of an architectonic system, means emphasizing the aesthetic, static and constructive value of a genial "tecnica aedificandi".

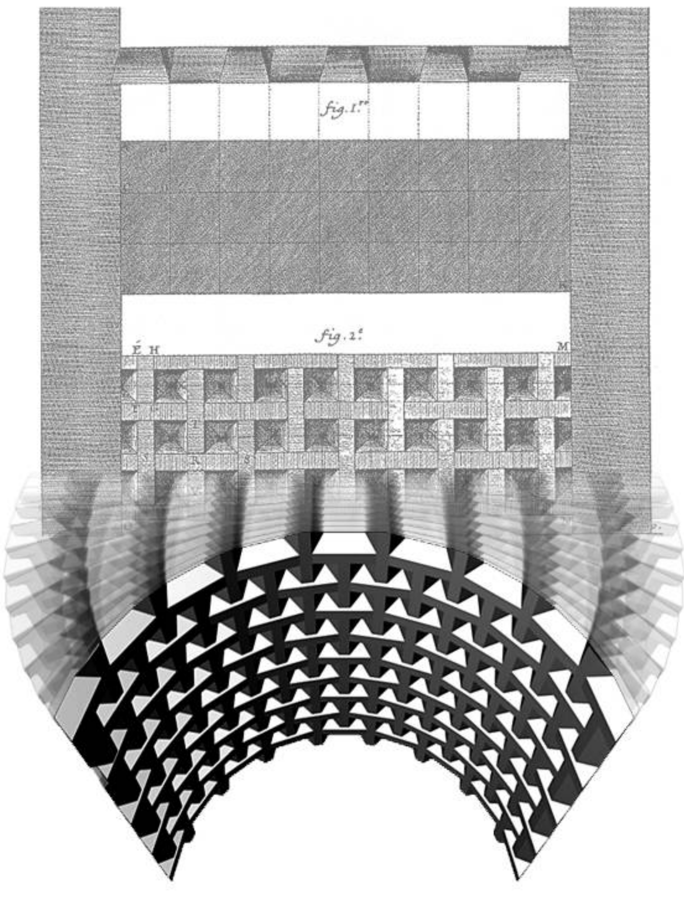

Figure 6. Transformations: Joseph Abeille's ashlar barrel valult.

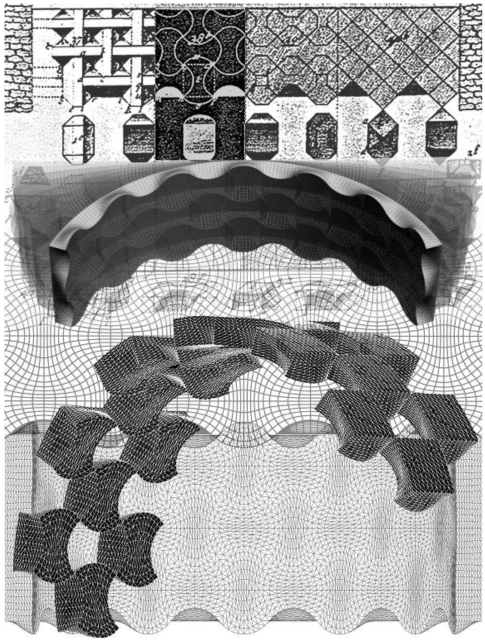

Figure 7. Transformations: Sébastien Truchet's ashlar barrel vault.

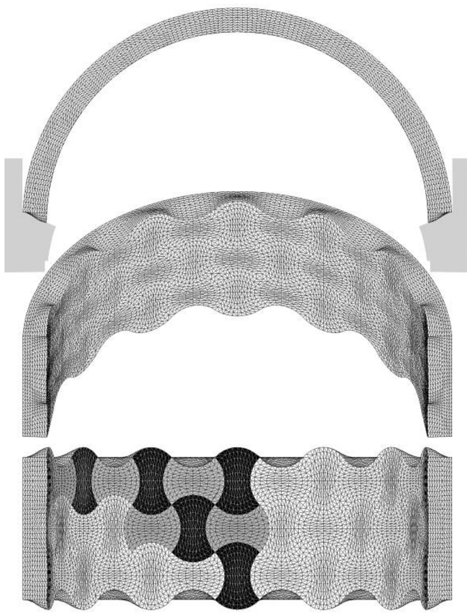

Figure 8. Transformations: Sébastien Truchet's ashlar barrel vault.

CONCLUSIONS

The results obtained are not exhaustive, as this is a recent experiment that tried to link together the syncretism of three-dimensional computer modelling with the morphological analysis of architecture. We draw some lines for the research, useful as well as unknown that should be developed in a more cross-curricular way. We propose a reading of stereotomic architecture "sub specie", from all possible angles. Such an interpretative category, if applied to the analysis of an architectural monument, is aimed at decrypting geometrical genesis and the dynamic processes required for the creation of a shape. But, it could go outside historic time limits finding today interesting as well with unexpected design potential. This would proof the vitality of stereotomic lessons projected into a new future.

ACKNOWLEDGEMENTS

Images have been developed with contributions from Marco Stefano Orsini, graduate of the Wood Stereotomy Laboratory, who did a research thesis about "*Carpintería de armar e carpintería de lazo" in Spain*. 2nd semester 2004-2005. Thesis supervisors, Professors Claudio D'Amato Guerrieri and Giuseppe Fallacara.

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