Saving the dome of St Peter's

Rowland Mainstone

The masonry dome of the pope's church of St Peter in Rome is one of the largest in existence. In spanning approximately 42.5 m it is comparable only with the domes of the Roman Pantheon and Florence Cathedral and it is partly matched in other respects only by the latter which must have served as the principal model for it. Growing fears for its stability in the early eighteenth century and the actions taken to allay them in 1742-48 would therefore be of interest however the fears had arisen and whatever the response had been. The fact that this response entailed the first attempts to apply new scientific tools of statical analysis to a major standing structure makes the whole episode of even greater significance in the history of construction. Moreover the attempts were unusually fully recorded and most of the records survive.

These attempts are therefore the main subject of the present paper. To set the scene we shall merely take a brief look first at the history of the dome's construction, at the subsequent fears, and at the state of scientific understanding in 1742.

Construction of the new church and its dome

Construction of a completely new church to replace Constantine's early 4th century timber-roofed one had begun in 1506 under the architect Bramante¹ The piers and linking arches destined to carry a vast central Pantheon-like dome were built in some haste over the next few years. But there followed almost a century of stop-go construction with repeated changes in design of both the church as a whole and the dome. Sangallo reinforced the piers to carry an even heavier dome as part of a more grandiose design. Michelangelo scaled this design back in a masterly manner and had the arches further reinforced before the drum that was to carry his dome was built in the late 1550s (Fig. 1). After another lengthy hiatus after his death an attic was added to this drum by della Porta and, between 1589 and 1590, he also built the dome itself to a slightly higher profile than Michelangelo had intended (Fig. 2).

Like that of Florence Cathedral the dome is double shelled over much of its height with internal meridional ribs (Fig. 3). It



Figure 1. The drum under construction in the late 1550s, seen from the south. Below it, a temporary structure incorporating the original apse of the Constantinian church protects the saint's shrine (drawing by Dosio).



Figure 2. The south-west exterior of the church seen from the Vatican gardens (author)





Figure 3. Elevation of the dome interior sectioned through the main supporting arches and plans at drum level and the level at which the shells divide (from Letarouilly, *Basilique de Saint Pierre*, Paris, 1882, plates 7 and 24).

differs in being circular rather than octagonal (Fig. 4) and, probably to speed construction, in having been built on centring, although this circularity would have made construction without centring much easier than it was in Florence.²

Over part of its height, the drum consists only of 16 piers separated by windows and linked by spur walls to coupled columns acting with these walls as outer buttresses. The piers are bridged above the windows by a bold entablature that projects forwards over the buttresses, and above this by the continuous ring of the attic and the lowest part of the dome. When the dome divides into two shells, the ribs effectively continue the piers upwards until they terminate in another continuous ring at the crown, which is surmounted by a lantern. Construction of the drum and buttresses is of stone with rubble fills and with spiral stairs inserted in four of the drum piers for access to the dome. This masonry gives way to brickwork in the dome. Two circumferential wrought iron eye-bar chains of about 75 cm² total cross section were built into the lower half of the dome as construction proceeded (Fig. 5 and lettered L,L in Fig. 8 and P,P in Fig. 9 and n,u in Fig.13).

Growing fears for the dome's stability and Vanvitelli proposals in 1742

Meridional cracking that probably developed as the centring was removed (if not before) would soon have been hidden on the inside by the setting beds for the mosaic decoration and by the lead sheathing of the exterior. It was again becoming noticeable at least by 1631, as must associated cracking of the drum and spur walls of the buttresses. By 1680 it had become sufficiently conspicuous to give rise to rumours that the dome was in danger.3 But the fears were then judged to have been exaggerated and no action was taken. Further concerns arose after an earthquake in 1730. In 1735 swallow-tail spies were inserted to monitor future movements. Matters came to a head in 1742 when Vanvitelli, the architect responsible for the church, prepared a report recommending various strengthening measures.4

Having examined both the dome and all its supports he concluded that there was nothing much wrong with the piers and their foundations. He also thought that cracks in two of the main arches spanning between them were of minor significance. He considered that



Figure 4. Looking down into the church from the dome showing the circular base for the drum (author).



Figure 5. Interior detail of the dome between the two shells at the level at which they divide. A short length of the original upper circumferential tie is exposed at the lower right a little beneath the surface of the inner shell. The slight thickenings of the shells just beyond this point and again in the distance mark the entrances to passageways through two of the ribs (author).

the trouble lay in the dome itself and the drum and buttresses. Its outward thrust had been too great for the adhesion of the mortar and the encircling chains.

To safeguard it in the future he recommended first the addition of three or four new chains suitably encased. They were to be supplemented chiefly by various reinforcements and consolidations of the masonry. No analysis was presented of the forces to be resisted and none seems to have been attempted.

There was nothing new in adding chains in this way. Similar additions had recently been made elsewhere, notably at the cathedral of Montefiascone and St Mark's in Venice.⁵The lack of quantitative analysis to justify the need and establish the necessary cross sections was also typical. But developments in theoretic understanding of how arched and vaulted structures behave and of structural stability in general did now open up the possibility of such analysis.

Structural understanding and possibilities of quantitative analysis in 1742

A brief outline of these developments must suffice for the benefit of any reader not already familiar with them.⁶

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The foundation of the new understanding was a precise generalised definition of the hitherto somewhat limited or nebulous concept of a force acting in any direction. It gave a new precision and power to existing understanding of how forces acted and reacted with one another and the effects that they had.

One approach was to consider them directly. When they met at a point and there was no resulting movement, any pair could be represented in both magnitude and direction by adjacent sides of a parallelogram and the third would then be represented by its diagonal or (what amounted to the same thing) all three could be represented by the three sides of a triangle. When, rather than so meeting, they acted in such a way at to tend to cause rotation about a point of support, it was necessary to consider the balance of their moments about that point (these moments being the products of the forces and the shortest distances from it of their lines of action). For equilibrium there should be no net moment.

The other approach was less direct. It considered the work that each would do if there was a notional very small displacement of the body on which they acted (this work being the product of the force and the displacement). For equilibrium the total work should be zero.

The first approach was the first to benefit from the new precision. In 1717 the second benefited similarly when Jean Bernoulli gave a definitive statement of it, describing it as the principle of virtual displacements though we now refer to it as the principle of virtual work. But it remained unpublished until 1725.⁷

Application of either approach to a practical structure called for a suitably simplified conceptual model, and devising this was not always easy. Up to 1742 only the first approach had been so applied.

The structural form that was easiest to model was the hanging chain in which there was an easily visualised transmission of tension along its curve from one end to the other. As Hooke had realised as early as 1670, there must be a similar transmission of compression from voussoir to voussoir in an arch – "*ut pendet continuum flexile sic stabit inversum contiguum rigidum*" as he later published the idea in anagram form.⁸ And in 1717 Stirling had explicitly modelled the voussoirs as frictionless balls. Consideration of the forces acting at successive points of contact from the crown downwards confirmed that these would follow an inverted catenary (Fig. 6).⁹ For design purposes however, this modelling was directly relevant only to the choice of profile, which was of less practical



Figure 6. Stirling's model of the arch as engraved for Poleni's *Memorie istoriche* (from plate D).

importance than the emphasis sometimes placed on it if (as usual) the arch ring had significant depth.

More relevant to the problem at St Peter's was the support requirement, in other words the horizontal thrust to be resisted. Early attempts by La Hire to calculate this for the arch were based on somewhat unrealistic models.¹⁰ In 1730 Couplet proposed a more realistic model." He based it on the observation that, when the supports began to move aside by rotating about the outer extremities of their feet, actual arches tended to give way by the hinge-like opening of joints in three other locations - on the extrados at the crown and on the intrados in both haunches. The five effective hinges in all reduced the arch to four rigid bodies whose equilibrium could easily be analysed by considering

directly the forces at the hinges and the balance of moments acting on the supports (Fig.7).

Nothing similar had yet been attempted for the three-dimensional structure of the dome, although the possibility of regarding it simply as a ring of arches tapering in width as they approached the crown had been seen by Hooke when he had the hunch in 1671 that the ideal profile was not a catenary (as it was for the simple arch of uniform cross section standing alone) but a cubic parabola.¹²

It was thanks to concern for St Peter's and to the then pope, Benedict XIV, that the attempt was made, and it was this attempt that first adopted the second approach.



Figure 7. Couplet's model of the arch with base hinges at R and F, a crown hinge at A, and hinges in the haunches at T and K (from *Mémoires de l'Académie Royale des Sciences*, 1730)

Commissioning of the three mathematicians and their Parere

As a humanist with wide contacts and some knowledge of what had already been achieved scientifically, the pope did not immediately accept Vanvitelli's proposals. Nor, as would probably have happened earlier, did he simply appoint a commission of Vanvitelli's peers to review them and either endorse them or hammer out a fresh consensus.¹³ Instead he commissioned an independent report from

three leading mathematicians (or scientists as we should probably now call them). They were Thomas Le Seur, François Jacquier, and Ruggiero Boscovich, all professors at the University of Rome and at the forefront of ongoing research. They reported after a remarkably short space of time on 8 January 1743.⁴⁴

In order to construct an appropriate model as a basis for analysis they sought first, by repeated detailed inspection, to learn as much as possible about the present state of the dome and all that it could disclose. They itemised and depicted on the best available drawings the damage they observed.15 This depiction must have served as the basis for fig.1 (the principal figure) of the plate appended to their report (here Fig. 8), though the cracking shown through the thickness at V near the springing level could only have been surmised from what they could see on the exterior. For comparison, Fig. 9 shows corresponding views and a partial plan from a slightly later much more comprehensive and precise survey.¹⁶ In the plan the crack that extends upwards from the crown of the annular vault of the access ramps and is lettered E in Fig. 8 is seen circling around the walkway above the vault.



CVPOLA DI S. PIETRO Figure 8. External and internal part elevations of the dome, the latter sectioned through the pendentives, showing cracks observed by the three mathematicians (from *Parere di tre mattematici*).



Figure 9. Similar part elevations to those in Fig. 8 with the internal one now sectioned through a main arch and a part plan at the level of the foot of the drum (from Vanvitelli's survey of the state of the dome in 1743 as engraved, somewhat simplified, for Poleni's *Memorie istoriche*, Stato de' difetti, plates II, XI and XVI).

Reflecting on the significance of the observed behaviour as a whole, they concluded, as Vanvitelli had done, that there was no risk of further damage from continuing settlements of the main piers and arches. They were much more concerned by the cracking in the dome, the way the relatively weak drum and buttresses were allowing it by spreading outwards at the top, and by indications (including broken spies) that the movements were continuing and not merely part of a harmless initial settling down.¹⁷ They thus saw the outward thrusting of the dome, weighed down also by the lantern, as the real threat to future safety. As evidence of this thrusting action, they referred to the damage to the spur walls of the drum buttresses and to the splitting of the base by the circumferential crack just referred to.18

While the radial cracking into lunes offered the possibility of a somewhat similar model to that adopted by Couplet, the complex threedimensional geometry and the much more extensive cracking precluded its direct adoption and made modelling difficult in four ways:

- In place of the crown voussoir or voussoirs of an arch there was an open ring and, above this, a lantern.
- 2) They found no evidence of hinging of the lunes in the haunch region. Lower down, horizontal cracks suggested hinging at several levels in the drum piers.
- 3) In addition to the crack through the base there were associated steeply inclined cracks in the spur walls above.
- 4) The lunes were not of uniform thickness, either throughout their height or circumferentially as rib and infill sections alternated. Nor were the drum and buttressing fully continuous circumferentially.

To cope with the first three difficulties, they modified Couplet's model as follows:19

They treated the crown ring of the dome in the same way as the crown voussoir of an arch.
In the absence of visible evidence of hinging in the dome above its springing level, they assumed a single hinge at this level (H in their small fig. 2 of the present Fig. 8).

3) They took into account the splitting of the buttresses and outer part of the base from the piers of the drum and inner part of the base by making separate analyses a) without it and b) with it ignoring resistance to relative displacement along the cracks.

Without the splitting they also ignored the evidence of hinging at other levels in the drum and considered all hinging of the unified support provided by attic, drum and buttresses to occur at A in their fig. 1 "or a little higher".

With it they considered all hinging of the attic and drum to occur at the level of the widow

sills (a in their fig. 1), again ignoring the evidence of hinging at other levels in the drum piers. They also made further assumptions that they described as allowing for the high compressions, but that are better seen as making some allowance for the actual hinging at higher levels in the drum. The principal one was that the hinge H would not rise as the support rotated under the thrust but would be displaced horizontally. Clearly this was intended to apply to the estimated movement of the centre of gravity of the dome. But it is less clear how it was assumed to affect the movements of the centres of gravity of the drum and buttresses and outer base. In relation to the contribution of the buttresses and outer base, they merely mention a "small addition" without specifying it.²⁰

4) Their ingenious way round the fourth difficulty and other three-dimensional complexities to was to consider the equilibrium not of individual lunes and their supports but of the threedimensional system as a whole. This took advantage of its overall radial symmetry if the differences between the dome ribs and intervening sections were ignored and likewise the circumferential variations in the support including the relative weakness of the four drum piers with internal stairs.

To assess this equilibrium they made virtual-work analyses for the entire system rather than considering directly the balance of internal forces.²¹ They estimated the virtual work that would be done *against* the chain tensions and *against* or by the rising and falling weights of lantern, dome, and their supports for a notional unit outward displacement at the level of the intermediate hinge H. Their calculated weights were for the whole ring of radial slices into which dome, attic, and drum were assumed to have split and their calculated inward radial forces exerted by the chains were the total forces around the circumference.

Striking the balance called merely for estimates of yield strengths, weights, and centres of gravity, followed by determinations of the relative displacements – radial for the chains and vertical for the weights. Chain tensions were estimated as strengths near failure – a reasonable basis for an analysis of the possibility of collapse – using Musschenbroek's test values.²² A further simple application of the virtual work principle showed them that the total extension of the chains would be $2\pi \times$ their radial displacements.²³

However they did not present their analyses in full on the grounds that "experts in geometry well versed in calculation would easily see for themselves how to proceed".²⁴ Moreover, just as they did not define precisely where they assumed the hinge positions to be, they did not define precisely the assumed line of the break between drum and buttresses for the second case, and they gave only some of their estimates of relevant weights and no indications of their estimated centres of gravity. They merely indicated very diagrammatically in their small fig.5 some of the other displacements that would take place as a result of the assumed displacement at the middle hinge and then tabulated their estimates of the forces exerted by the weights and the chains for a unit displacement there.

The present Figs. 10 and 11 are therefore only tentative reconstructions, incomplete in Fig. 11 because of the greater uncertainties there about some of their estimates and assumptions.²⁵ Since they did not fully specify their assumed hinge positions, numerous similar analyses were made for other possible positions. Those reproduced are the ones that led to the best overall agreement with the displacements implied by their estimated forces. These displacements cannot have been scaled directly from large-scale plots of finite displacements.²⁶ They could have been arrived at only as the ratios of two measurements. It was therefore assumed that the actual measurements were those lettered a, b, c, d and x, y in the smaller explanatory drawings for rotations about the relevant instantaneous centres of rotation. If this assumption is correct, it was probably the first major occasion when the idea of the instantaneous centre was put to practical use.²⁷

Without splitting of the buttresses and outer part of the base from the piers of the drum and inner



Figure 10. Reconstructed virtual-work analysis by the three mathematicians assuming no separation

of buttresses and drum (author).

Figure 11. Reconstructed virtual-work analysis by the three mathematicians assuming a separation of buttresses and drum (author).

part of the base, the forces in M libbre (1 libbra = 3.3N) for a unit displacement at the central hinge were:²⁸

the two chains	2.674919 stabilising
the rising attic, drum, buttresses and base	18.373475 stabilising
the falling lantern	1.853235 destabilising

Even without taking into account the further stabilising action of the lifting of the dome as its base was lifted by the outwardly rotating support, the stabilising action of the chains and the rising weight of drum, buttresses and base exceeded by such a large margin the destabilising action of the falling lantern that they did not even estimate the action of the dome. Had this further action been included the excess of stabilising action would have been about 27M libbre (90MN). With the splitting, the forces were:²⁹

he two chains	2.674919 stabilising
he rising attic	0.867444 stabilising
he rising drum	1.266690 stabilising
he buttresses	0.574555 stabilising
he outer part of the base	0.752686 stabilising
he falling lantern	2.961060 destabilising
he falling dome	6.412590 destabilising

The estimated stabilising action thus fell short of the destabilising action by more than 3M libbre (10MN). They argued that this shortcoming indicated a possible risk to future safety and took it as

the basis for determining what additional inward tie force was desirable at the intermediate hinge level. They also considered that, to insure further against accidents, it was desirable not only to tie the dome in this way to prevent it thrusting outwards but also to tie its support to enable it to resist any thrust. They therefore proposed new chains at the springing level, around the drum cornice, and around the base, all of 62 cm² cross section, the first to prevent thrusting and the others to resist a possible thrust. In addition, but now without giving their reasons, they recommended chains at the base of the lantern and mid height of the dome, plus reinforcement of the buttressing and the making good of existing cracks.³⁰

Initial reactions to the Parere and the Three Mathematicians' Riflessioni

Whether the pope was alarmed by this assessment or merely wary it is difficult to say. He responded by ordering the immediate publication of the *Parere* and seeking other views before a decision was reached, notably by commissioning a second opinion on it from another eminent scientist, Giovanni Poleni, a professor at the University of Padua.

There were also meetings in Rome to discuss the report, at which further views were contributed in addition to written submissions. The three mathematicians' analyses were largely ignored. Nearly all emphasis was on diagnosis of the damage suffered and suggested remedies. Numerous contributors saw poor hasty construction, differential settlements, the great weight of the dome and lantern, alternating heat and cold, or occurrences like earthquakes and lightning, as prime or contributory causes of the present damage. But not all looked beyond them, as the three mathematicians had done, to identify aspects that might get worse and should be prevented from doing so. Nevertheless about half did agree with Vanvitelli and the three mathematicians that excessive thrust constituted the chief risk and called for additional ties to resist it.³¹

In response to the discussion and on the basis of two further inspections – but without yet having seen Poleni's comments on their *Parere* – the three mathematicians submitted a further report, their *Riflessioni.*³²

In this they clarified some aspects of the *Parere* and further justified their conclusions about the nature of the hazard and its cause. In particular they insisted that the continuing damage to the dome did not stem from settlement of the piers or cracking of the main arches because there was no sign of continuing movement there. In response to the obvious objection that the dome should have collapsed already if their second analysis were correct, they made it clear that the calculated shortcoming there of the stabilising action ignored the resistance to relative displacement along the separation cracks between drum and buttresses. In other words, though they did not say so, it presented what might now be referred to as a lower bound worst case for stability. It was because this resistance could not with certainty be estimated or relied upon that they saw the shortcoming as indicating the best force to ensure future safety.

Poleni's Riflessioni and Aggiunta

Of all the views presented it is Poleni's that are of greatest interest however. These therefore are the only ones that will be considered further here. Working solely on the basis of the *Parere* and other material sent to him in Padua, and apparently without having seen the three mathematicians' *Riflessioni*, he first gave his views in a lengthy manuscript dispatched to Rome on 21 March – his own *Riflessioni*.³³

Although he found their model ingenious, he had difficulty in envisaging the actual dome behaving in the same way. Since it was built of mortared masonry and not timber or metal, he would expect considerable horizontal cracking above its springing level.³⁴ He also doubted whether



Figure 12. Poleni's small model of

of his Riflessioni as redrawn for

plate H of his Memorie istoriche).

the collapse mode visualised by the three mathematicians (from Fig. 1

the ribs could behave as one with the intervening webs.³⁵ To test their model's behaviour, he made his own small physical model of the whole dome and drum and cut it radially into four sections and each section into two at the springing level (Fig. 12). He found that in this model the rotations assumed by the three mathematicians about the intermediate and base hinges could not occur without large openings which would lead to very high local stresses at the points of contact in the real structure. Ignoring the fact that there was a vast difference between splitting into four sections (as in his model) and into a far greater number, he saw this as a further reason for rejecting the three mathematicians' model.³⁶

Praiseworthy as he found their approach, he therefore doubted its validity.

More generally, he expressed the view that, although mathematical theory was useful in design [*per fabbriche*], it was less useful for analysing a damaged standing structure: diversities of form, materials, and manner of construction lead to such

variations in behaviour that it provided too uncertain a basis for speculating on the causes of damage. It was necessary to argue directly from the facts.³⁷

After reviewing current theory in more detail than the three mathematicians had considered necessary and discussing the observed damage he gave his own assessment of its significance. But, like others, he concentrated on its possible early origins and paid less attention to continuing causes that could lead to a progressive worsening.³⁸ He did not see the dome as being in immediate peril. Nevertheless he agreed with the three mathematicians about the desirability of new chains and gave his own initial recommendations for sizes and locations, though without giving any basis for them.³⁹

On receipt of these *Riflessioni*, he was invited to Rome to see the situation for himself. This he did, apparently accompanied by Vanvitelli who had already embarked on the major new survey referred to earlier (Fig. 9).

In the much shorter *Aggiunta* ⁴⁰ which he then wrote to supplement his *Riflessioni*, he stated that he found no evidence of damage to the piers, little to the arches, and still considered the dome in no present danger. He nevertheless agreed that deviations from the perpendicular and cracks in the drum and dome would worsen with time and lead eventually to grave danger if nothing was done. He therefore now recommended four new chains, each with a cross section of 52 cm² in the positions marked A, B, C, D in Fig.13, and gave full specifications for their fabrication and installation.⁴¹ Again there was no supporting analysis. He merely stated that they would be larger than the existing chains.

Installation of new ties and Poleni's Memorie istoriche

It was these recommendations that were finally accepted, apparently without further reference to the three mathematicians. Vanvitelli was entrusted with installing the chains and with other desirable works. Chain installation was completed in 1744 while work continued on making good earlier damage, and included that of an additional one at the base of the lantern (at E in Fig.13) after further lightning damage.⁴² When, four years later, it was found that the upper original chain was broken, another one was added a little below it at Z in Fig. 13. This largely completed the restoration.⁴³

But it was not quite the end of the story.

After the first two new chains had been installed, Poleni was commissioned to write the account of the whole episode that has already cited several times as a source – his *Memorie istoriche*. This



Figure 13. Positions of the original circumferential chains (n and u) and added chains (A to E and Z). The manner of installation of the added ones is shown in the detail (from *Memoric istoriche* plate K).

was published four years later as a large folio volume of well over 200 pages of closely printed text in double-columns with 28 further pages of annotated fold-out engraved illustrations.

It is indeed the principal single source for the entire episode from the origins of the basilica to the conclusion of the remedial works. But it is somewhat confusingly arranged and is not quite the comprehensive straightforward history that might have been expected. In particular it not only fails to give an adequate summary of his *Riflessioni* in book III: it also fails to indicate precisely what his own subsequent contribution to specifying the remedial works was. His justification for these shortcomings is that he had already included descriptions of all his tests and diagnoses in the latter part of book I.⁴⁴ These descriptions are the principal value of the book for the present purpose. We therefore turn to them next, merely noting that none of them was undertaken to indicate desirable chain strengths and that the most reasonable conclusion in the absence of any dating is that all were undertaken only when he was back in Padua while installation of the chains proceeded. He must have relied more than he admits on the three mathematicians' analyses and on Vanvitelli's judgement in specifying what was to be done, in spite of earlier having dismissed the analyses.

He describes four sets of tests of which brief mention should suffice for all but the first. Of these latter tests the second set was on thermal expansion.⁴⁵ The third was an extensive series on iron bars to give a better basis than Musschenbroek's tests for estimating the strengths of bars of particular cross sections.⁴⁶ And the fourth was made merely to confirm approximately of the truth of the three mathematicians' conclusion that the total inward force exerted by a circular chain would be $2\pi \times$ the direct tension (Fig. 14).⁴⁷



Figure 14. Poleni's model to measure the total inward force exerted by an approximately circular tie. The hanging weights oppose the inward forces and thereby measure them. (from Memorie istoriche plate F)

The first test was the only one directly relevant to assessing the dome's stability. Being familiar like the three mathematicians with the work of Hooke, La Hire, and particularly Stirling on arch behaviour, but again preferring experiment to calculation and more justifiably in this instance, he made a modified hanging chain model to determine a possible line of thrust through it. Ignoring his earlier objection to considering the webs as acting integrally with the ribs, he modelled an arbitrary 50th slice of the whole dome as a chain whose links carried lead weights proportionate to the weights of equivalent solid voussoir-like sections into which he notionally divided it.48 Finding that the chain, suitably supported, assumed a curve that lay wholly within Vanvitelli's cross section when it was inverted (Fig. 15 left), he now used this fact to support his earlier view that the dome was not in itself in danger⁴⁹. But surprisingly he left it at that.

Looking back

Looking back with our present much wider knowledge we see can see very clearly the difficulties of assessing the safety of a standing structure like the dome. In designing an as-yet unbuilt structure, the designer is free (within the limits of practicality) to choose whatever characteristics he would like it to have.⁵⁰ But in assessing a standing structure its complex present damaged state must be the starting point and a valid analysis must allow adequately for all its relevant characteristics. Without all our present knowledge, and without our present means of supplementing this by more revealing measures of changes taking place, the difficulties of doing so were much magnified.

The three mathematicians and Poleni approached the task very differently.

The three mathematicians' approach

The three mathematicians did focus throughout on the problem of assessment in the light of all they could discover about the present state of the dome. Having identified the main hazard, and with no precedent to guide them, they took the imaginative leap from the highly complex reality that confronted them to a simple model that was amenable to static analysis based on recently developed theory.

This model had somehow to recognise both the partial break between the drum and its buttresses and the multiple hinging. Since the support given to the drum by the buttresses could not be assessed with any certainty, it was reasonable to perform two separate analyses bracketing the possibilities - one ignoring the break and the other for a complete break with the drum. Ignoring the break was highly optimistic and unsafe. Assuming in the second analysis that there was no resistance to relative translation was definitely on the safe side. But by what margin it would still be difficult to say without knowing more about the associated assumption about the displacement of the buttresses that was hinted at but not clearly specified. And the gap between the two analyses was so large that it left only the second as a guide.

The ingenious assumption in this analysis of no lifting of the dome by the outward rotation of the drum was also almost certainly a safe recognition of the actual hinging of the drum that was otherwise ignored. But it is impossible to say by what margin without better information on the hinging. Vanvitelli's survey, showing fewer cracks than the three mathematicians' fig.1, suggests fewer hinges and a large margin.



determination of a thrust line. IeuxyV, in the dome, ignoring the ties. Whether intentionally or not it is roughly the line for minimum thrust passing through the crown hinge position indicated by cracks in the ribs. Right at a: the same thrust line extended to the base of the drum. Right at b: the corresponding thrust line deflected inwards by the original ties and similarly extended to the base of the drum (from Memorie istoriche plate E with author additions on the right).

Most doubts arise from the choice of hinge positions in the model.

To start with the less important which had no major effect on the outcome, all hinges should really have been a little in from the faces of the masonry. The lowest should have been at the mean floor height of the access ramps (eight in all from the four entries over the pendentives rising to exits over the main arches) and not at the entry level as seems to have been assumed. And the top one should have been considerably higher than they show it in their small fig.2 and as it is seen in the present Figs. 10 and 11. It should have been more in line with the crack lettered OP in their fig.1 or the mean position of the same crack as shown in Vanvitelli's detailed elevations of all ribs.⁵¹

The level of the intermediate hinge is more critical and presents a greater problem. Despite the actual multiple hinging, there could only be one representative hinge for analysis to proceed. But it should not have been located at the springing level.⁵² Even with the top hinge relocated, a single hinge there implies a thrust line between the two that would pass well below the dome soffit over most of its height. A higher location seems essential but it would have meant that there would have been more intermediate hinging to be allowed for somehow.

Poleni's approach

To these extents, Poleni's scepticism about the use of mathematical theory for analysing a damaged standing structure was justified on this occasion. But he failed to devise an alternative.

He seems to have been distrustful of calculation and to have preferred to remain the experimental scientist. While he advanced the science by his tests of strengths and thermal expansion and by his adaptation of Stirling's model of the arch, he failed to make the full imaginative leap from the complex reality of the problem faced to seeing the full potential value of this model.

If only he had further developed it by extending the chain up to a point corresponding to the base of the drum he could have obtained a complete thrust line down to the base (Fig. 15 right a). Better still, he could have included the tie chains, representing their inward actions by attaching suitably weighted horizontal cords. This would have provided an alternative basis (Fig. 15 right b) for an overall assessment. Or he could, by similar experiment, have explored a wider range of tying possibilities. Judgement would still have been called for in deciding how close the thrust could safely approach the boundaries of the chosen representative cross section. In one sense it would have been more clear-cut than that called for by the three mathematicians' approach. But this would be so only if other implied judgements about such things as the validity taking this cross section as a basis were not made explicit.

The present situation

We thus see that neither approach was wholly successful. For first attempts, with no precedents as a guide and confronted by such a difficult structure, this is hardly surprising. But the three mathematicians did point the way ahead.

Even today, with much more powerful analytical techniques at our disposal, we must still model complex situations as much simpler ones. Judgements must still be made and the choice of analytical technique and model should be as conducive as possible to proper judgement.³³

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Correspondence: Dr. Rowland Mainstone, 20 Fishpool Street, St Albans, AL3 4RT.

References

- The whole history of the reconstruction is outlined further in R. J. Mainstone, 'The dome of St Peter's: structural aspects of its design and construction and inquiries into its stability', *AA files*, 39 (1999) pp. 21-39 (earlier version with fewer illustrations in idem. Structure in Architecture: history, design and innovation, (Aldershot, 1999), pp. 327-350).
- For Florence see idem, 'Brunelleschi's dome', Architectural Review, vol. 162 (1977), pp. 156-166 (reprinted in Structure in architecture, pp. 327-350) and for St Peter's see the paper cited above, pp. 26-30.
- 3. F. Baldinucci, Vita del Cavaliere Gio. Lorenzo Bernino (Florence, 1682), pp. 82-102.
- 4. Vanvitelli's report, Breve parere di N. N., is summarised in G. Poleni, Memorie istoriche della gran cupola del Tempio Vaticano (Padua, 1748) (referred to below as Memorie istoriche), paragraphs 455-460.
- 5. Memorie istoriche, paragraphs 183,187.
- 6. These developments are now best dealt with more fully in E. Benvenuto, An introduction to the history of structural mechanics, 2 vols (New York, 1991).
- 7. P. Varignon, *Nouvelle méchanique ou statique* (Paris, 1725), vol.2, pp. 174-189, in which it was introduced as a corollary of the triangle of forces.

- 9. J. Stirling, 'Methodus disponendi quotcunque spheras in fornicem, et inde demonstratur proprietas praecipua curvae catenariae', *Lineae tertii ordinis Newtonianae* (Oxford, 1717), appendix pp. 11-14.
- 10. P. de la Hire, 'Sur la construction des voûtes dans les édifices', Mémoires de l'Académie Royale des Sciences, 1712, pp. 91-101.
- P. Couplet, 'De la poussée des voûtes', Mémoires de l'Académie Royale des Sciences, 1730, pp. 107-163.
- 12. J. Heyman, 'Hooke's cubico-parabolical conoid', Royal Society Notes, 52, (1998), pp. 39-50.
- For earlier instances see R. J. Mainstone, 'Structural theory and design before 1742', Architectural Review, 143, (1968), pp. 303-310 (reprinted in idem, Structure in architecture: history, design and innovation, pp. 81-109).
- 14. T. Le Seur, F. Jacquier and R. G. Boscovich, Parere di tre mattematici sopra i danni, che si sono trovati nella cupola di S. Pietro, (Rome, 1743) (referred to below as Parere).
- 15. Parere, p. 25. The most likely sources are the drawings that served as a basis for the plates on pages 313 and 323 of C. Fontana, *Templum Vaticanum et ipsius origo* (Rome, 1694), and plate 25 in P. Bonani, *Numismata summorum pontificum templi Vaticani fabricam indicantia* ... (Rome, 1696), both of which show similar dome profiles and are similarly sectioned through the pendentives, although the three mathematicians' section misleadingly shows the vault of the access ramp within the base as rising throughout to the height it reaches only above the main arches. Later surveys, beginning with Vanvitelli's and including Letarouilly's, show a steeper profile.
- 16. Engravings of almost the whole series of these survey drawings appear as plates I-XIX of *Memorie istoriche* with copious facing annotations of the cracking and out-of-plum measurements of the drum piers. Selected original drawings are illustrated in R. di Stefano, *La cupola di San Pietro* (Naples, 1963), figs 39-40, 42, 44-47; rev. edn, 1980, figs 54-58, 61-72, and in idem, 'Luigi Vanvitelli ingegnere e restauratore', Luigi Vanritelli (Naples, 1973), figs 414-428.
- 17. Parere, p. 13.
- 18. Parere, pp. 15-19.
- 19. Parere, pp. 23-24, 28-29.
- 20. The relevant text reads 'Giudichiamo pero di non dilungarci molto dal vero, se consideriamo le piegature tutte come raccolte e unite in una sola al fondo delle finestre ivi appunto, dove in a si vede slamato lo stipite, e tale la compressione tanto esteriore ne'medesimi siti, quanto interiore nell'imposta, che nella Fig.5 il punto H della linea CH non debba elevarsi sopra l'oriziontale QH, ma rimangasi al suo livello. In tal caso coll'aiuto dell'esposto problema, parte con una piccola aggiunta, che convien farvi per determinare il momento de'contraforti staccati, e della parte esterior della base, troviamo che equivale la spinta.'
- 21. Parere, p. 26.
- 22. Parere, pp .25-26; P. van Musschenbroek, Introductio ad cohaerentiam corporum firmorum, (Leyden, 1729).
- 23. Parere, pp. 26-27.
- 24. Parere, p. 27.
- 25. For a less detailed account of another attempted reconstruction, see G. M. López, *Estabilidad y construcción de cúpulas de fábrica: el nacimento de la teoría y su relacion con la práctica*, thesis, University of Madrid, 1998, pp. 228-230.
- 26. This is betrayed by the listing of weights, chain strengths, and forces all to six, seven or eight figures. Had the displacements been scaled directly it could have been to two significant figures at most and, where the corresponding weights etc. are given to allow checking, they could not have led to the precise forces listed.

- 27. The first published reference appears to be 'De centro spontaneo rotationis' among the ANEKDOTA of volume 4 of Johann Bernoulli's works, *Opera Omnia*, (Lausanne and Geneva, 1742), pp. 265-273. But, as the prefatory notes in volume 1 indicate, he seems to have sanctioned the inclusion of these previously unpublished items only reluctantly and to have given the editor no assistance in arranging them. Hence the date when this piece was written is not stated. It could have been considerably earlier and the idea could have been around soon after that of the virtual-work approach to analysing equilibrium.
- 28. Parere, pp. 28-29.

29. Parere, pp. 29-30.

30. Parere, pp. 30-35.

- 31. Summaries of all reports not already referred to or in what follows appear in *Memorie istoriche*, paragraphs 253-271, 292-323, 354-362, 384-438, and 461-517. Much briefer summaries appear in R. Di Stefano, *La cupola*, pp. 36-55; rev. edn, pp. 26-41, and more selectively in G. M. López, 'Teoría de estructuras y restauración en el siglo XVIII: la cúpula de San Pedro de Roma', *OP Ingeniería y Territorio*, III (2001), pp. 48-59.
- 32. T. Le Seur, F, Jacquier and R. G. Boscovich, Riflessioni sopra alcune difficoltà spettanti i danni, e rifarcimenti della cupola di S. Pietro, (Rome, 1743).
- 33. G. Poleni, *Riflessioni di Giovanni Poleni sopra i danni e la ristaurazioni della cupola del tempio di S, Pietro in Roma*, Padua, 21 March 1743, MS Cod. Marc. It. IV, 658, Venice (referred to below as Poleni, *Riflessioni*).
- 34. Poleni, Riflessioni, paragraph 9.
- 35. Poleni, Riflessioni, paragraph 10.
- 36. Poleni, Riflessioni, paragraphs 11-17.
- 37. Poleni, Riflessioni, paragraph 30.
- 38. Poleni, Riflessioni, paragraphs 26-60 passim.
- 39. Poleni, Riflessioni, paragraphs 65-82 passim.
- 40. G. Poleni, Aggiunta alle riflessioni di Giovanni Poleni sopra i danni e la ristaurazioni della cupola del tempio di S, Pietro in Roma, Rome, 10 June 1743, MS Cod. Marc. It. IV 659, Venice, published with only minor omissions in Memorie istoriche, pp. 323-330.

41. Aggiunta, paragraphs 9-12.

- 42. Memorie istoriche, paragraphs 572-603.
- 43. Memorie istoriche, paragraphs 605-663.
- 44. Memorie istoriche, paragraph 364.
- 45. Memorie istoriche, paragraphs 99-106.
- 46. Memorie istoriche, paragraphs 141-145.
- 47. Memorie istoriche, paragraphs 146-153.
- 48. Memorie istoriche, paragraphs 85-87.
- 49. Memorie istoriche, paragraph 88.
- 50. E. M. Gauthey, for instance, had an easier task in modelling the as-yet unbuilt dome of the Paris Pantheon in *Mémoire sur l'application des principes de la méchanique à la construction des* voûtes et des dômes, (Dijon, 1771).
- 51. Memorie istoriche, plates XVII-XIX.
- 52. In placing it there the three mathematicians must have ignored the fact that open cracks on the outer surface at a higher level would have been hidden by the lead sheathing.
- 53. Although the focus is different, much of R. J. Mainstone, 'Structural analysis, structural understanding, and historical interpretation', *Journal of the Society of Architectural Historians*, 56, (1997), pp. 316-339 (reprinted in idem. *Structure in Architecture: history, design and innovation*, Aldershot, 1999, pp. 123-147) deals with essentially the same choice today in situations similar to that at St Peter's