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Construction History, Vol. 3, 1987

55

Innovation in Structural Theory in the Nineteenth Century*

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At the beginning of the nineteenth century the state of knowledge was such that theory of structures was ripe for rapid development. This was due, for example, to the work of the Bernoullis, Euler and Coulomb. Thus elementary statics was being applied to masonry arches and timberwork and the elastic theory of bending of beams (still in common use) had been determined. The principle of virtual velocities (nowadays termed virtual work), of ancient origin, was available as a powerful aid in the application of statics. Construction for which statics was insufficient, in that supplementary conditions relating to elastic deflexions had to be introduced for precise analysis, though not uncommon was not understood. For this reason calculations were approximate. Included in this category were the encastré timber beam and the beam supported at intermediate points as well as at each end (the continuous beam). The former was acknowledged, notably by Robison before 1800, to be twice as strong as the same beam simply supported at its ends; the latter was probably regarded as a succession of encastré spans for estimating its strength. Timber framework was used at that time for bridge and roof trusses using design principles established by practice over the years [1].

Rapid development of theory of structures in the nineteenth century was undoubtedly stimulated by the emergence of the railway era and metal construction. However, some major advances were premature and probably due to the keenness of individuals. In this respect the Frenchman C. L. M. Navier (1785–1837) affords an outstanding example [2]. Thus the value of his general method of finding the forces in the bars of loaded frameworks, including those with bars or elements supernumary to the needs of statics, was not widely recognised for more than a century. But his theory of *encastré* and continuous beams was soon accepted. It was used to great advantage in Britain, notably by Brunel and Robert Stephenson through their assistants, Bell and Wild respectively, who were made aware of it by Moseley [3]. That theory was, perhaps, the principal advance in relation to engineering practice before 1850 and was refined subsequently. It is an example of the theory of statically-indeterminate structures based on the assumption of elasticity (the elastic theory) with which much innovation in the nineteenth century was concerned.

The nature of structural forms over the years had been dictated by the available materials of construction and their properties. The arch and pillar which utilise the

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56 Innovation in Structural Theory

compressive strength of stone or cast iron, and the beam and latticework which utilises the strength of timber in both compression and tension, were dominant before 1800. At that time wrought iron strong in both tension and compression was in its infancy, but it was soon to assume great importance in affording new and economical structural forms such as plate and tubular girders for heavy duty. For example, Telford used wrought iron for his Menai suspension bridge in 1826.

Although Moseley was familiar with Coulomb's theory of the masonry arch, still unsurpassed according to Heyman [4], he and others sought to refine it. But Brunel was satisfied with the method of elementary statics [5] in the 'wedge theory' variously ascribed to Hooke, De La Hire, Parent and David Gregory, for which the ultimate strength theory of Yvon Villarceau provided implicit justification. Moseley's outstanding book *The Mechanical Principles of Engineering and Architecture* of 1843, described as the first modern engineering text in English, included not only theory of masonry structures but, of great importance, an account in detail of Navier's theory of beams including continuous beams. Consequently Moseley's followers, notably W. Pole, became involved in the first major structures in wrought iron which were being conceived by the railway engineers.

Unfortunately at about the same time (1847) a trussed iron girder bridge at Chester collapsed during the passage of a train with fatal consequences. The bridge consisted of cast-iron girders trussed with wrought-iron rods to prevent tension being developed in the cast iron whose tensile strength is low, using a concept similar to that in reinforced concrete which was to appear much later. A government inquiry into the safety of iron bridges for railways followed and the Royal Commission (1847–48) for a time seriously threatened progress in iron bridge construction, in wrought iron as well as cast. (This was according to the discussion of W. Fairbairn's paper on tubular girder bridges (1850) [6], of which he was patentee.)

Nevertheless, designs for Brunel's Chepstow railway bridge and Stephenson's monumental bridge over the Menai Straits proceeded using Navier's theory as introduced by Moseley. Stephenson's bridge was especially difficult to design and led to further innovation relating to continuous beams by the adoption of a kind of prestressing. Navigation requirements dictated no more than four spans, side spans of 230 feet and centre spans of 460 feet with constant height and lateral space. Wrought iron tubular construction was chosen, the dimensions of the tubes being such that the railway could pass through them. The bridge was to consist of a pair of continuous girder beams with a single line railway through each. The tubes were rectangular in section and made of rivetted wrought iron plate, a technology developed earlier for ship construction. The self-weight of the beams would have been too great for safety without some means of prestressing. The problem was resolved by erecting and joining by rivetting over the supports, the individual spans of tube, using jacks to introduce predetermined misalignment prior to jointing. When the joints were made and the structure restored to normal support levels, the bending moments due to self-weight were equalised at mid-span and support points. This was an effect which could be achieved alternatively by making the joints and adjusting the levels of the intermediate supports afterwards by jacking and inserting 'shims'. The effect was almost as though the whole length of tube had been placed on the five supports simultaneously. Without that prestressing process, the erection and jointing of the individual spans would have been insufficiently advantageous because the full mid-span effect of the (greater) self-weight, as for a simply supported span, would have remained. It was said to have been the first time Navier's theory had been used for such a large structure-and

probably the last time also! It was the subject of much international interest and criticism.

Navier's theory of continuous beams was refined by Clapeyron and others, but the Englishman Heppel [7] almost anticipated Clapeyron and his 'theorem of the three moments'. This Heppel did while working alone in India on a bridge project. Curiously the American A. Jay Du Bois [8] asserted that continuous beam theory was unknown in America and Britain until 1870. He credited his fellow countryman Merriman with its introduction to the English-speaking world. That inaccuracy was compounded in 1875 when the *Philosophical Magazine* published Merriman's work after its publication in America and after having some years previously published Heppel's contribution!

By 1850 the lattice girder and bar truss for roofs and bridges had appeared, including the Warren girder or truss, notably for the Newark Dyke Bridge of the Great Northern Railway. The theory, consisting of the conditions of equilibrium of the joints, had been used for more modest timber structures much earlier, by Robison for example whom Navier (1826), Cotterill [9], and Weyrauch [10] acknowledged. It is believed that Jourawski in Russia and Whipple in America wrote treatises on the subject in 1847.

Graphical methods for this and other engineering problems were not well known until later (c.1860). On the continent those methods were widely exploited and taught by Culmann and others while in Britain Rankine [11] and Maxwell [12] developed them extensively for triangulated trusses. Nevertheless, according to R. H. Bow [13] (of Bow's notation fame), Stephenson's assistant C. H. Wild had used a force diagram for a simple truss before 1854. Cotterill in an article which appeared in 1869 used the funicular polygon method for constructing bending moment diagrams for a beam [14]. The antiquity of simple graphical statics is indicated by the belief, on evidence of a publication of 1725, that the mathematician Varignon devised the funicular polygon c.1687. Cotterill, however, thought it was discovered by Robison c.1805. Clapeyron and Lamé are believed to have published a memoir on graphical statics in 1826, as well as one on the theory of arches in 1823 when they were youthful engineers employed by the Russian government on design and construction of iron suspension bridges. Notwithstanding these developments graphical analysis (in which Navier seemed to show no interest) was to be transitory and is now extinct for practical purposes.

The theory of the elastic arch, of timber or wrought iron, was developed by the Frenchman Bresse c.1854 [15] though Navier had dealt with the essence of the problem some thirty years earlier in his theory of the two-pin elastic arch rib. This was another problem of statical-indeterminacy. Interest in the theory of the masonry voussoir arch was revived by the temptation of the apparent precision of the elastic theory assuming the elasticity of masonry. Navier had seemed to suggest it, but now the aging J. V. Poncelet supported the theory strongly, but without success. Poncelet was an important and distinguished authority on applied mechanics generally. He is especially noted for introducing the concept of conservation of energy into practical mechanics c.1831 particularly with regard to work and strain energy. Moseley used that theory in 1843 in relation especially to problems of elastic deflection.

With the availability of a wide variety of rolled sections in wrought iron, lattice and truss construction became widespread especially for railway bridges. Trusses which were statically-indeterminate (i.e. with members supernumary to the needs of statics) for practical convenience, appeared. An example was the type used successfully by Thomas Bouch assisted by Bow (notably for the Belah viaduct in 1860) using approximate theory. Soon precise methods of analysing such frameworks attracted

58 Innovation in Structural Theory

attention. Navier's method was overlooked from the outset; Moseley did not foresee its possibilities, for he omitted it in 1843. Essentially the equations of equilibrium of the joints of such frameworks had to be supplemented by those specifying the compatability, geometrically, of the small elastic strains of the connected bars, on the basis that connections were maintained after the application of load. This was an essential and obvious requirement for any structure. Clerk Maxwell (1864) [16] the physicist (like Moseley) and German engineer O. Mohr (1874) [17] were the leaders in solving the problem. They proposed, independently, a method which was identical in principle and the inversion of Navier's method. Fleeming Jenkin, Maxwell's engineer friend, attempted to simplify the details of the method in 1869 [18] by the use of the principle of virtual work. This was the same device as that which Mohr was to use to great effect in his version of the method. Nevertheless, the contributions of Maxwell and Jenkin were unknown in Europe for many years.

At about the same time attempts were being made in Italy by Castigliano after Menabrea to develop the method using a device based on strain energy rather than virtual work. The idea probably stemmed from an eighteenth-century interest in the concept of natural economy already embodied in certain principles such as that of 'least action' (which has not survived) to enable problems not apparently amenable to statics to be resolved. One example was the problem of a solid supported at four points, which Euler attempted to solve without success. In that tradition L. F. Menabrea [19] made a proposal about the equations necessary to supplement those of statics (those of strain compatability) for the solution of a statically-indeterminate framework. He proposed that such equations could be obtained readily by the conditions specifying that the strain energy of the framework is a minimum with respect to the forces in the statically supernumary bars. This he did as early as 1858. The law of conservation of energy relating the strain energy of a framework (the sum of the strain energies of the bars) to the work done by the loading had been established earlier notably by Clapeyron and Poncelet. Though his applications of the principle were successful, his proof of its general validity was inadequate and was challenged by Castigliano some 15 years later [20]. Castigliano was followed after about another ten years in an independent derivation of that principle by the German Fränkel [21] whose work was particularly elegant. But the principle tended to be known as Menabrea's principle on the continent and Castigliano's principle of least work in Britain. Cotterill's independent contribution of 1865 [22] is noteworthy. Among the examples chosen by Fränkel to demonstrate the use of the principle was the stiffened suspension bridge. Thereby he provided by chance the elastic theory of suspension bridges, in advance of Lévy whose exhaustive study of the subject from first principles earned for him credit for the theory. It was also the subject of independent derivations by W. Ritter and H. Müller-Breslau. The theory was later to be superseded by the "deflexion theory" of the Frenchman, Godard which took account of the important contribution of gravity to the stiffness of the more flexible and economical bridges (formerly these bridges were designed merely by statics). (In passing it is appropriate to acknowledge Lévy's work in theory of structures generally. It includes a modified version of Navier's original method for bar frameworks and a study of parallel chord trusses to determine which form was conducive to least weight of material. He found in favour of the Warren truss. His celebrated book La Statique Graphique of 1874 [23] deals inter alia with these aspects.) Maxwell derived a theory of minimum weight frameworks c.1870 using the principle of virtual work implicitly, but it was apparently too abstract to interest engineers.

Innovation in the theory of structures towards the end of the century also included adaptation of the principles discovered to the analysis of forces and moments in rigidly jointed frameworks. Examples included rigidly jointed trusses and portal frameworks. The former had hitherto been considered to be pin jointed, and axial forces in the bars or members were determined to the exclusion of the bending (secondary) stresses induced in the bars as the framework suffered deflexion, however small. Those stresses were small so long as the bars were slender; to avoid them by actually using pin joints would be unduly expensive and generally impracticable.

A development of great importance was the systematisation of the analytical process for finding the forces and bending moments in structures generally by introducing the concepts of flexibility and stiffness coefficients. The latter were used by the mathematician Clebsch [24] a disciple of Navier, as long ago as 1862. In the simplest terms flexibility relates to deflexion caused by unit applied force while stiffness relates to the force on a structure which causes unit deflexion at a point subject to specified conditions. Those concepts without which structural calculations would be unmanageable nowadays, have been fully exploited in the present century in relation especially to the use of computers and in this respect Navier's method has finally attained supremacy.

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Construction History, Vol. 3, 1987

Agents of Change: Hennebique, Mouchel and ferroconcrete in Britain, 1897–1908*

PATRICIA CUSACK

This paper discusses some of the circumstances of the introduction and establishment of reinforced-concrete framing in Britain [1]. Reinforced concrete existed by the early 1900s as a collection of patented, and a few unpatented, 'systems', with varying dispositions of reinforcement; some systems were designed only for specific structural elements, such as floors or pipes, while others were adapted, not always appropriately, to entire building frames. These systems were commercially exploited by their patentees, but technical details were guarded from public knowledge and protected by vigorous litigation. In 1904, there were over 50 such systems [2]. Among the most flexible and the most widely used was François Hennebique's system, developed in Belgium and France and extended world-wide through a specialist commercial and technical organisation, which was in turn imitated by other major system specialists such as Coignet [3].

The introduction of reinforced-concrete framing in Britain at the turn of the century was a result of Hennebique's business policy of international expansion. Existing commercial exchanges between Nantes, where Hennebique had an agent, and Swansea, may have facilitated the contacts which led to the commissioning and erection in 1897 of the first fully framed and entirely reinforced-concrete building in Britain, Weaver & Co.'s provender mill in Swansea [4]. During the construction of the mill Hennebique selected a General Agent for his system in Britain, L. G. Mouchel, whose work, until his death in 1908, effectively established reinforced concrete in Britain and especially its use for framed buildings, albeit as a private, commercial product.

This commercial aspect, also the existence of competing methods and self-interest, prompted architects and others to start investigations into reinforced concrete, to broaden professional interests in it and to end the specialists' monopoly [5].

Reinforced Concrete at the Turn of the Century

By the mid-1890s, a number of companies and individuals in Britain were employing reinforced concrete (some since at least the 1860s), but primarily for floors, roofs and beams and probably not for framed construction. Some of these companies, including W. B. Wilkinson & Co., the pioneer patentees of reinforced concrete (1854-55),

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